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$$2\mu t = \lambda.$$

$$\therefore t = \frac{\lambda}{2\mu_f}.$$

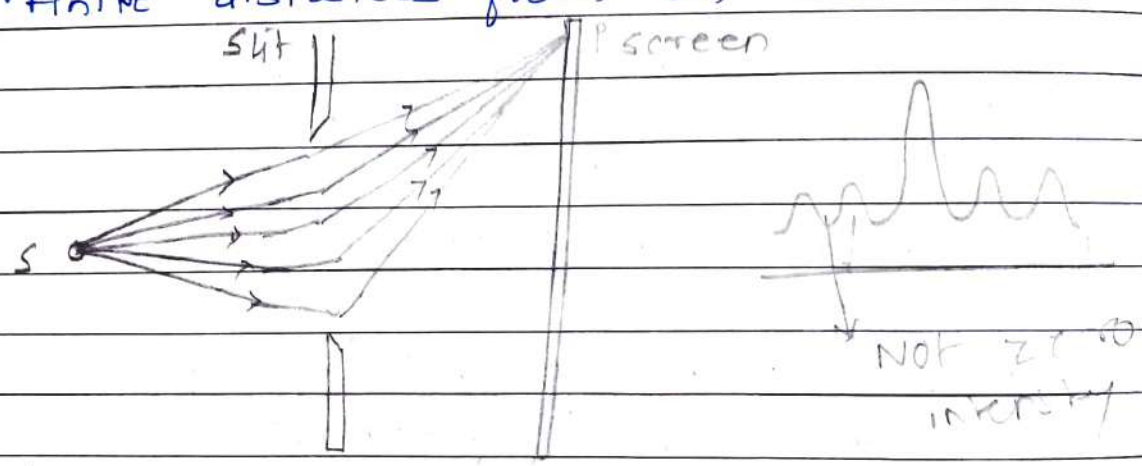
& also $\mu_f = \sqrt{\mu_m}$.

~~2, 3, 5, 7, 20, 25, 48, 52~~ ..

Preeset 1-

Diffraction:- Encrochment of light through an obstacle is known as diffraction.

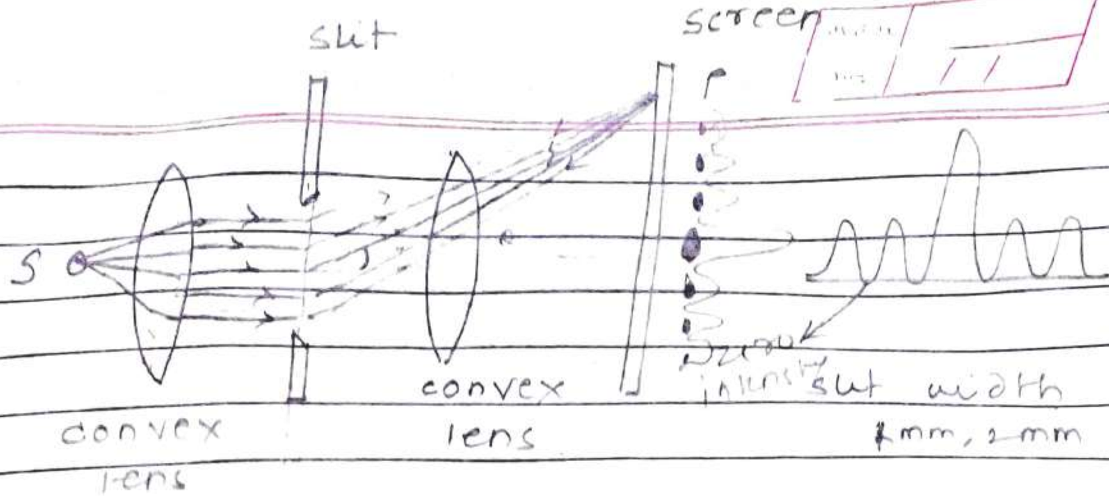
Fresnel diffraction:- In this type, the source of light and the screen are effectively at finite distances from an obstacle.



1 Lenses are not used to make the rays parallel or convergent.

2 incident wavefront is not planar, it is spherical or cylindrical

3 It is experimentally simple but the analysis is very complex



Fraunhoffer Diffraction :- In this type of diffraction, the source of light & the screen are effectively at infinite dist--ances from the obstacle.

1. The conditions required for this diffraction are achieved by using two convex lenses - i) one make the light from source parallel & ii) other to focus the light after diffraction on the screen

2. incident wavefront is planar. & the secondary wavelets which originate from unblocked portions of the wave front, are same in phase at every point in obstacle.

3. Diffraction is produced by the interfer--ence between parallel rays, by which brought into focus with the help of convex lens.

4. problem is simple to handle & analyse.

Fres	Fraun
1 Finite dist	1 infin
2 spherical or cylindrical	2 planar
3 No convex	3 [illegible]

Fraunhoffer Diffraction at a single slit

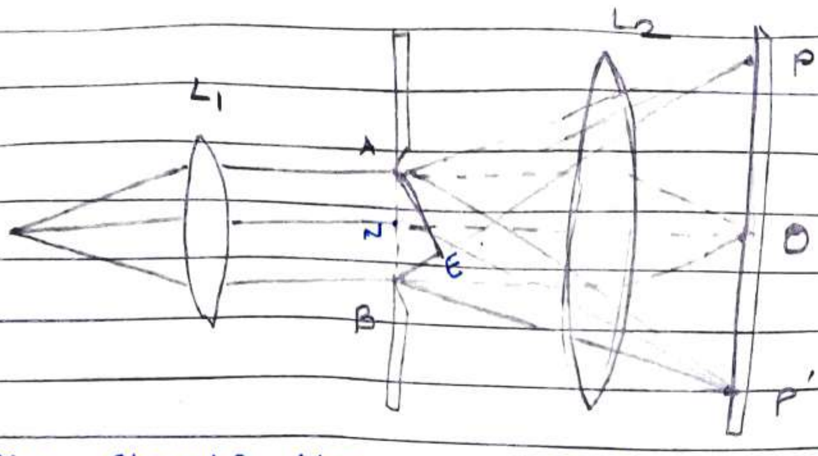


Fig shows the parallel wavefront incident on the slit AB

* According to Hygen's principle, each point on AB acts as a source of secondary wavelets, we ~~will~~
* All the points on AB are in phase & coherent.
* Hence the light from one portion of the slit can interfere with light from another portion & the resultant intensity on the screen will depend on direction θ
* The rays travelling from center of point source ~~let say~~ 'O', travels same distance in reaching 'O'. The optical path difference is ~~there~~ therefore zero and waves reaching at 'O' will be in phase. They reinforce each other to produce maximum intensity

Now consider the secondary waves travelling in a direction making an angle θ with ON

The secondary wavelets are brought to focus by lens at a point P, which may have maximum or minimum intensity depending on path diff. between the waves arriving at 'P'.

The path diffrence between these waves
$= BE$

$\therefore$ $BE = AB \sin\theta = e \sin\theta$
[$\because$ AB = slit width = e]

By definition
phase difference = $\frac{2\pi}{\lambda} \times$ path difference

$= \frac{2\pi}{\lambda} \times e \cdot \sin\theta$

As slit is divided into 'n' parts.
and each part can act as an elementary source
Hence,
phase diff $= \frac{1}{n}\left(\frac{2\pi}{\lambda} e \sin\theta\right) = d$ (let say)

According equations of resultant amplitude.

nslit

$$R = \frac{a \sin\left[\frac{\text{phase diff.}}{2}\right]}{\sin\left[\frac{\text{phase diff}}{2}\right]}$$

$$= \frac{a \sin\left[n\frac{d}{2}\right]}{\sin d/2} = \frac{a \sin\left[\frac{n}{n} \cdot \frac{2\pi}{\lambda} e \sin\theta\right]}{\sin\left[n \frac{2\pi}{\lambda} e \sin\theta\right]}$$

$$= \frac{a \sin\left[\pi/\lambda \, e \sin\theta\right]}{\sin\left[\frac{n\pi}{\lambda} e \sin\theta\right]}$$

let $\frac{\pi}{\lambda} e \cdot \sin\theta = \alpha$

$$\therefore R = \frac{a \sin\alpha}{\sin(\alpha/n)}$$

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as α/n is too small

$\therefore \sin \alpha/n \approx \alpha/n$

$$\therefore R = \frac{n \cdot a \cdot \sin\alpha}{\alpha}$$

if $n \to \infty$ then $a \to 0$

But $a \neq 0$.

$\therefore na = \text{const} = A_0$ (let say)

$$\therefore R = A_0 \frac{\sin\alpha}{\alpha}$$

The intensity $I \propto |A_0|^2$

$$\therefore I = A_0^2 \left(\frac{\sin\alpha}{\alpha}\right)^2.$$

* condition for Minima

$\sin\alpha = 0$

$\because \alpha \neq 0 \Rightarrow \alpha = n\pi$

$\therefore$ conditions for minima $\pm n\pi$
or $\pm n\lambda$

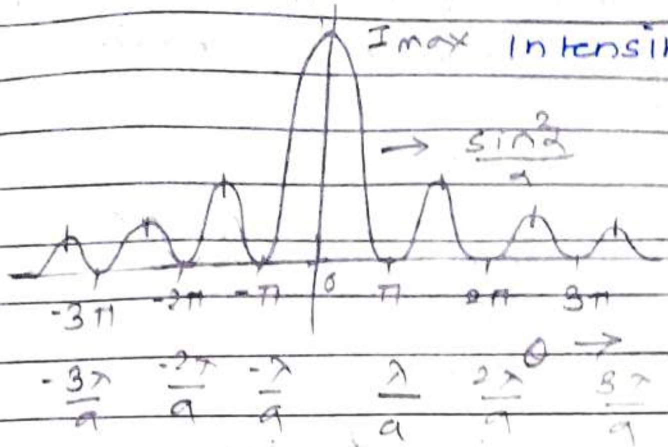
$$\therefore \frac{\pi}{\lambda}(a\sin\theta) = \pm n\pi$$

$$\text{or } \sin\theta = \pm \frac{n\lambda}{a} \quad \left\{\begin{matrix} n = 1, 2, 3 \ldots \\ \neq 0 \end{matrix}\right\}$$

& Maxima

$$\sin\theta = \frac{(2n+1)}{2}\lambda = \frac{(2n+1)\pi}{2}.$$

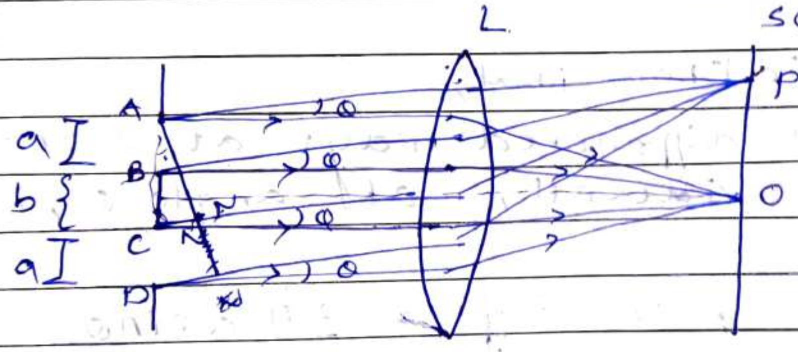
$$\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2} \ldots$$



Intensity distribution curve

The term $A_0\left[\frac{\sin^2\alpha}{\alpha^2}\right]$ gives the central maximum at $\theta = 0$

Fraunhofer Diffraction due to double slit



Let AB & CD are two rectangular slit parallel to one another.
The width of open slits = a
& " of closed slit = b
In this case the diffraction has to be considered in two parts.
i) The interference phenomenon due to secondary waves emanating from the corresponding points of two slits
2) The diffraction pattern due to two slits individually.

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let us consider the secondary waves travelling a direction inclined at an angle θ

in ΔCAN

$$\sin\theta = \frac{CN}{AC} = \quad (\because AC = (a+b))$$

$$= \frac{CN}{(a+b)}$$

& CN = path diff.

$\therefore$ path diff $= (a+b)\sin\theta$

Now from defination.

phase diff $= \frac{2\pi}{\lambda}(a+b)\sin\theta$

$= 2\beta$ (let say) — ①

We know that,

* The diffracted rays at corners A & B individually at angle θ, reaching P

$\therefore$ path diff. $= \frac{2\pi}{\lambda} a\sin\theta$

$= 2\alpha$ — ②

Resultant Amplitude

$$R = A_0 \frac{\sin\alpha}{\alpha} \quad — ③$$

Now the resultant amplitude due to effect of second slit B

phase diff $= 2\beta$

$$\boxed{R = 2A_0 \frac{\sin\alpha}{\alpha}\cos\beta}$$

As $I \propto |A_0|^2$

$$\therefore I = 4 A_0^2 \frac{\sin^2\alpha}{\alpha^2} \cos^2\beta$$

Thus the resultant intensity in the double slit pattern depends on

1. $\frac{A_0^2 \sin^2\alpha}{\alpha^2}$ → Gives diffraction band similar to single slit

2. $\cos^2\beta$ → Gives system of interference fringes due to superposition of two waves

Maxima & Minima

1. central Maximum at $\theta = 0$

& minima is

$\sin\alpha = 0$

$\therefore \alpha = \pm n\pi$

$\sin\theta = \pm \frac{n\lambda}{a}$ $\left[\begin{array}{c} n = 1, 2, 3 \dots \\ n \neq 0 \end{array} \right]$

only one slit Maxima

$\sin\theta = \frac{(2n+1)\lambda}{2}$

$\therefore \alpha = \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}, \pm \frac{7\pi}{2} \dots$

2. According to second factor $\cos^2\beta$

effect of both:

Maxima will be at $\cos^2\beta = 1$

when $n = 0, 1, 2, 3 \dots$

we know

$2\beta = \frac{2\pi}{\lambda} (a+b) \sin\theta$

$\beta = \frac{\pi}{\lambda} (a+b) \sin\theta$

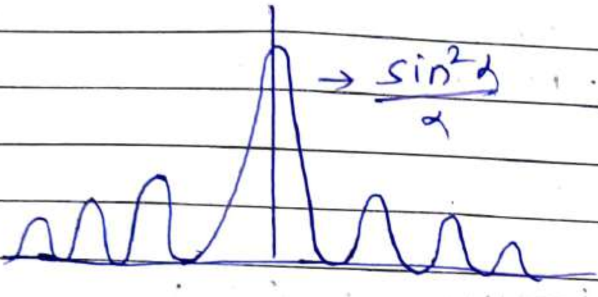
$\frac{\pi}{\lambda}(a+b)\sin\theta = \pm n\pi$

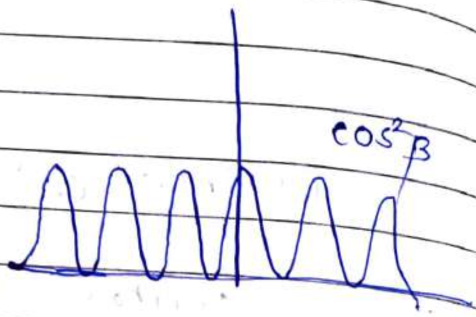
$= \pm n\lambda$

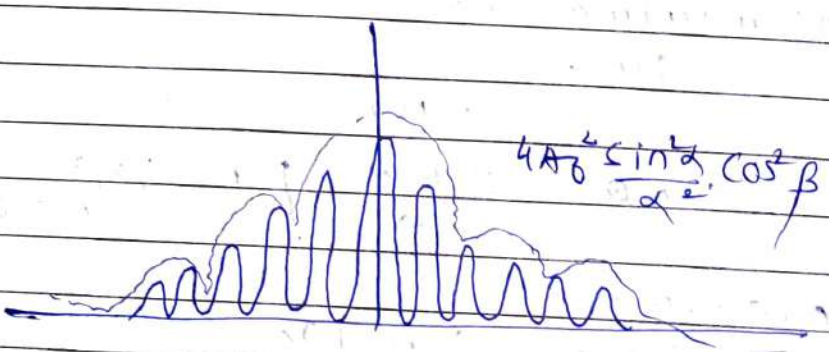
when $n = 0$, $\theta = 0$.

Thus central maxima lies along the direction of incident light called principal maxima of zeroth order.

Hence







* Missing order - When the light is diffracted, it can also produce an interfer. pattern e.g. with two slits, the two pattern superimpose for interfere. pattern appears within diffraction pattern envelope. Hence diffraction minimum may appear at same place as an interfer maximum & so cancel it out, giving.

Diffraction Grating

It consist of a very large number of extremly narrow parallel slits seperated by opaque surfaces.
There are two types of grating
(i) Transmission grating through which light is transmitted
(ii) Reflection grating from which light is reflected.

The transmission gratings are produced by ruling extremely close equidistant & parallel lines on optically plane glass plates with diamond point. The rulings (diamond scrach) scatter light and are effectively opaque while other parts without ruling transmit light & acts as slits.

Grating Element

If the width of each ruling is 'a' and the width of each slit is 'b' The length (a+b) is called 'grating element'

As there large of No. of lines say N lines, So (N-1) slits are present

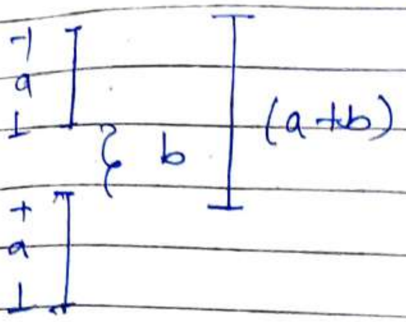
$\therefore$ Number of lines = N ≃ No. of slits (N-1)

$$\therefore \text{No. of lines/cm} = \frac{1}{(a+b)}$$

(a+b) is called 'grating const' or Grating period.

no. of lines/inch $= \frac{2.54}{(a+b)}$

$\because$ 1 inch = 2.54 cm



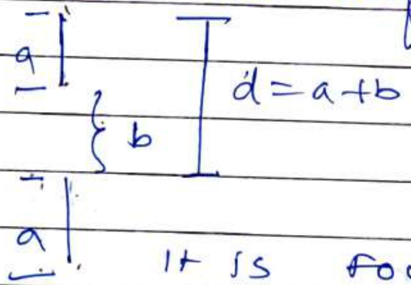
$$(a+b)\sin\theta = \pm n\lambda$$

$(n = 0, 1, 2, 3 \ldots)$

the above expression is known as "grating law", for the principal maximum

Missing orders : Absent specra.

Sometimes a few maxima disappear from the diffraction pattern



It is found that for a grating of opaque space with width b, slit width a, $d = (a+b)$

condition for maxima

$(a+b)\sin\theta = n\lambda$ — (1)

single slit diffraction ← $a \sin\theta = m\lambda$ — (2)

(1) / (2)

$$\frac{a+b}{a} = \frac{n}{m}$$

$$\therefore n = \frac{(a+b)}{a} m$$

(1) The above is the condition for n^{th} order spectrum absent.

if ~~n = 2m = 2 (∵ m∈N)~~ a = b
then m = 2n.

$$\left\{ \begin{array}{l} \frac{2a}{a}(m) = n \\ \cancel{2m = n} \end{array} \right.$$

Hence regular order of minima are n = 1, 2, 3 - - - .
& maxima = 2, 4, 6 will be missing. means all even order will be remain absent.

(2) if a = 2b. then m = 3n.

substituting regular orders of minima n = 1, 2, 3 - - - here it is found that m = 3, 6, 9 - - - - will remain absent.

Hence absent spectra depend on dimensions of the grating.

<u>Highest possible order</u>

The condition for maxima.

$$(a+b)\sin\theta = n\lambda$$

For monochromatic light λ incident on grating (a+b)

$$\therefore (a+b)\sin\theta_{max} = n_{max}\lambda$$

$$n_{max} = \frac{(a+b)}{\lambda}$$

if $\sin\theta_{max} = 1$

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→ 1) If the grating element is less than wavelength λ.

ie. $(a+b) < \lambda$

Then $n_{max} < 1$

→ 2) If grating element is less than double the λ

ie. $(a+b) < 2\lambda$

Then this means central maximum & first two order maxima are visible

$$\therefore (a+b) = \frac{1}{\text{No. of lines/cm}}$$

$$\therefore n_{max} = \frac{1}{\lambda \times \text{no. of lines/cm}}$$

* Therefore larger the no. of rulings smaller the number of visible orders.

Resultant intensity

$$I = A_0 \underbrace{\left[\frac{\sin \alpha}{\alpha}\right]^2}_{\text{single slit}} \underbrace{\left[\frac{\sin (N\beta/2)}{N\sin\beta/2}\right]}_{\text{interference of 'N' waves}}$$

Q → 2nd term → Max^m

1st " → zero

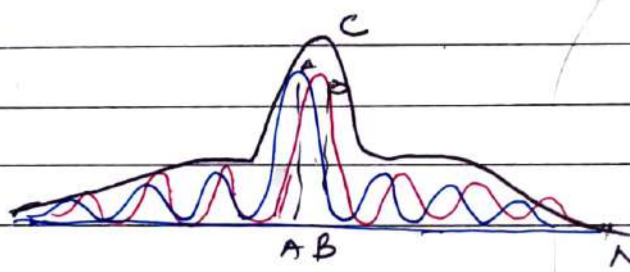
$I = 0$

Resolving Power

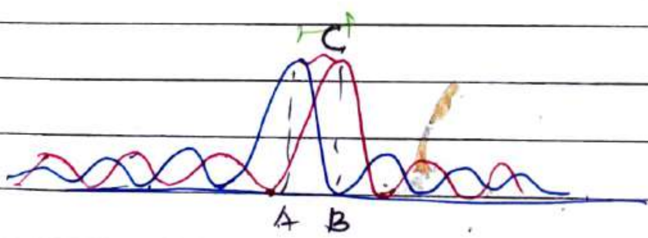
The method adapted to seeing the close objects as seperated images of two located very close to each other is called "Resolution".

& the ability of an optical instrument to produce distinctly seperate images of two objects located very close to each other is called "Resolving power".

It is also defined as reciprocal of the smallest angle subtended at objective by two point objects which can be distinguished as seperate.

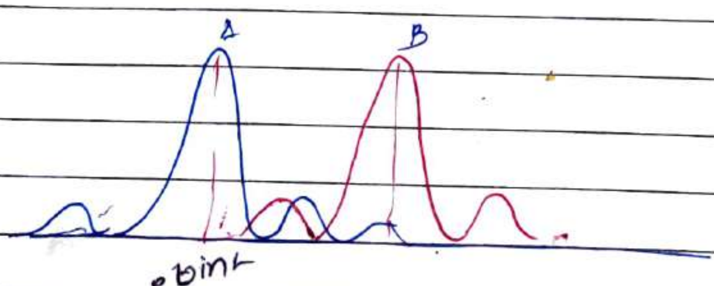


Not Resolved



Just resolved.

$d = 1mm$ min^m dist. of resolution



Totally Resolved.

Two point sources of light are said to be ~~set~~ resolved by an optical instrument only if 1st principal maxm of diffraction due to one source coincides with the first minm of diffraction due to other source

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conditions for the m^{th} order maxima for waves of 'λ' & $\lambda+d\lambda$ which are diffracted through θ & $d\theta+d\theta$ are

$$(a+b)\sin\theta = m\lambda \quad \text{—— (1)}$$

$$(a+b)\sin(\theta+d\theta) = m(\lambda+d\lambda) \quad \text{—(2)}$$

The distance between central maxima due to wavelengths λ & $\lambda+d\lambda$ is

$$P_1P_2 = (a+b)\sin\theta(\theta+d\theta) - (a+b)\sin\theta$$

$$= m(\lambda+d\lambda) - m\lambda$$

$$= m\,d\lambda$$

From the equation of 'N' of slits conditions for minima is

$$N \boxed{(a+b)\sin\theta = \frac{n\lambda}{N}} \quad (\text{where } n = 0,1,2,3 \ldots)$$

$$\therefore (a+b)\sin\theta = \frac{\lambda}{N}, \frac{2\lambda}{N} \ldots\ldots$$

where $(a+b)\sin\theta = 0$ is corresponds to central Maxima

∴ Hence it is

$$P_1P_2 = m\,d\lambda = \frac{\lambda}{N}$$

∴ Resolving power of grating

$$m\,d\lambda = \frac{\lambda}{N}$$

$$m \cdot N = \frac{\lambda}{d\lambda} = RP$$

It is defined as the ratio of any spectral line to the diff in wave-length.

$\frac{\lambda}{d\lambda}$ = $\frac{n}{1}$, 2, 3

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between the former & neighboring line such that two lines appear just resolved.

'N' is minimum number of lines on grating surface to resolve wavelengths λ & $\lambda + d\lambda$

- increasing number of 'N' on grating surace, gives improved R.P.

Dispersive power

Dispersive power of grating is defined as the ratio of difference in angle of diffraction of any two neighboring spectral lines to the difference in their wavelengths.

OR

The difference in angle of diffraction to the change in wavelengths.

$$(a+b)\sin\theta = n\lambda \qquad -(1)$$

differentiating w.r.t x

$$(a+b)\cos\theta \, d\theta = n\, d\lambda \qquad -(2)$$

$$\therefore \frac{d\theta}{\ } \quad \frac{\cancel{(a+b)\cos\theta}\, d\theta}{\cancel{(a+b)\sin\theta}}$$

$$\therefore \frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}$$

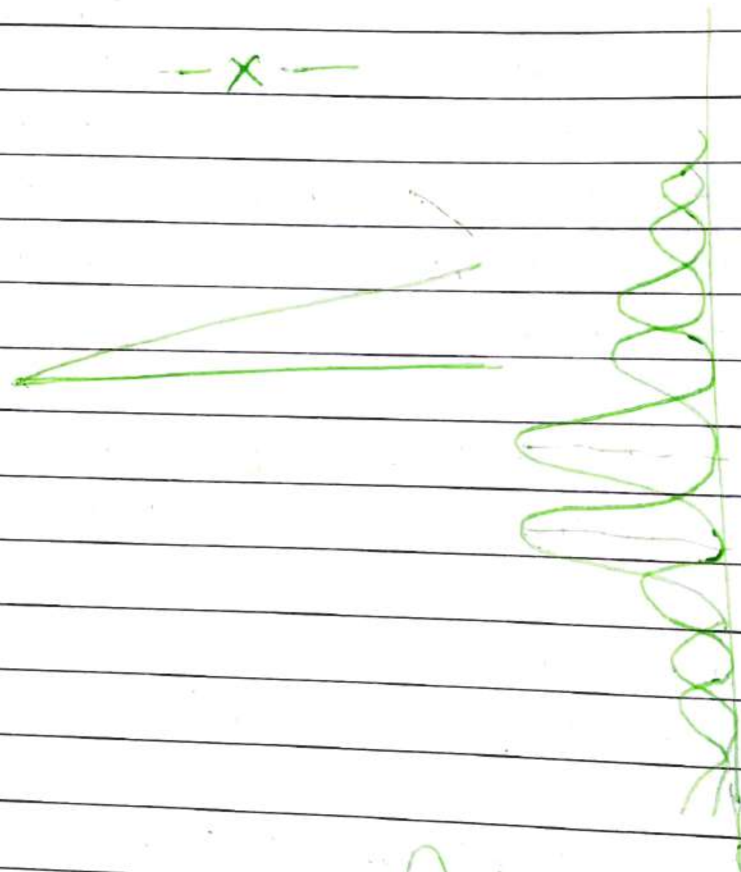
$$\frac{d\theta}{d\lambda} = \frac{nN}{\cos\theta}$$

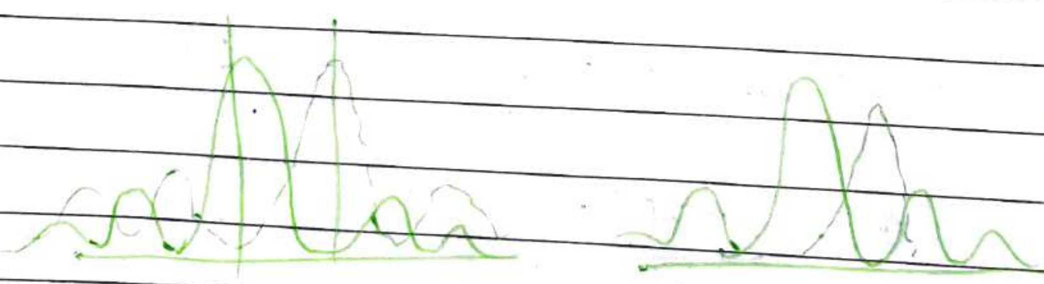
1) it is directly prop to order of spectrum 'n', i.e. for higher order dispersive power is more

2) it is directly prop. to 'N' no of lines on grating. Hence angular disp. is more with large no. of grating lines

3) it is inversely prop to $\cos\theta$ for $\theta = 0°$ $\cos\theta = 1$ & dispersive power is min^m

-x-





FIBRE OPTICS

2.8 Introduction

Fibre optics is a technology in which information is transmitted from one place to another with the help of an optical signal propagating through optical fibres. Optical fibres are used to transmit light signals over long distances.

An optical fibre is defined as a dielectric waveguide that confines light energy to within its surface and guides it in a direction parallel to its axis.

2.9 Principle of Fibre Optics : Total Internal Reflection

- The optical beam is made to travel through the optical fibre not by the simple mode of transmission but by the principle of total internal reflection.
- Whenever a ray of light comes from a rarer medium (of refractive index μ_1) and enters a denser medium (of r.i., $\mu_2 > \mu_1$) it bends towards the normal as shown in

Fig. 2.19 (a). In this case, the angle of refraction, $\theta < i$, the angle of incidence and Snell's law is written as

$$\frac{\sin i}{\sin \theta} = \frac{\mu_2}{\mu_1} > 1 \quad \text{.................... (2.10-a)}$$

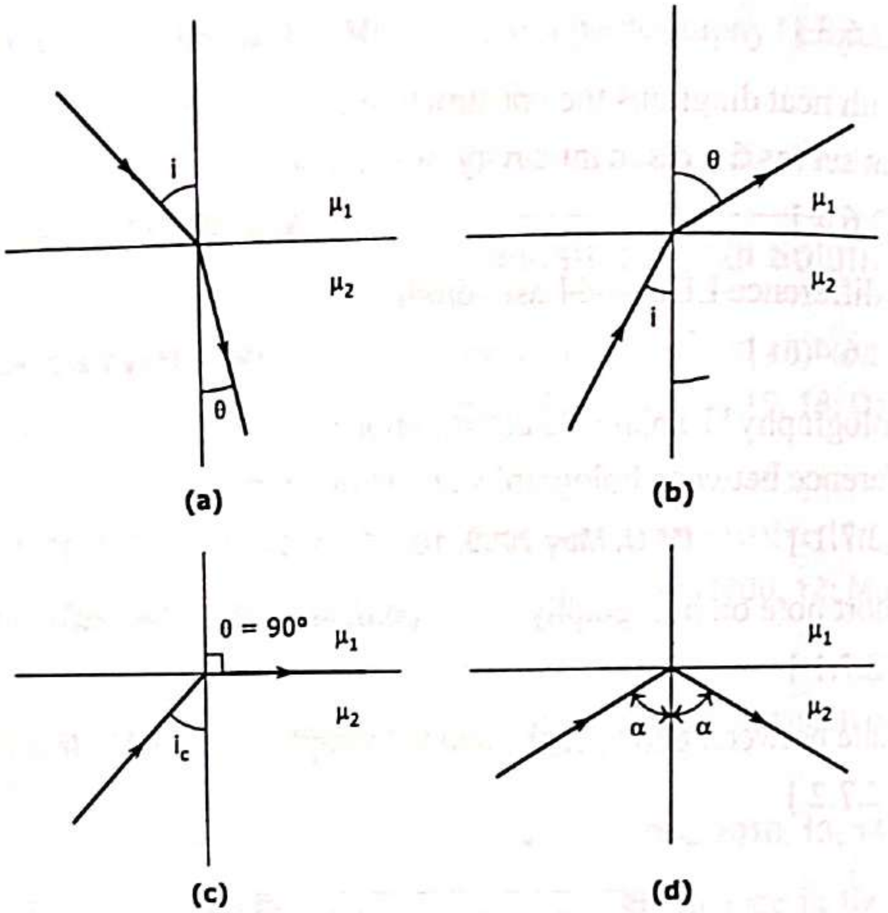


Fig. 2.19

- On the otherhand, if a ray of light falls on a denser surface after passing through a rarer medium, the refracted ray bends away from the normal on the interface [Fig. 2.19 (b)] In this case

$$\theta > i$$

and Snell's law becomes

$$\frac{\sin i}{\sin \theta} = \frac{\mu_1}{\mu_2} < 1 \quad \text{.................... (2.10-b)}$$

- Now, if the angle of incidence, i is gradually increased, the angle of refraction, θ also increases and a time comes when θ becomes equal to 90° [See Fig. 2.19 (c)]. The angle of incidence for θ = 90° is called the ***critical angle, i_c***. In this case, Snell's law is written as

$$\sin i_c = \frac{\mu_1}{\mu_2} < 1 \quad \text{.................... (2.10-c)}$$

- Finally, if a ray of light in denser medium is incident on the interface at an angle of incidence, $i > i_c$, the critical angle the light is reflected back into the denser medium [See Fig. 2.19 (d)]. This reflection is termed as ***total internal reflection***. The minimum angle of incidence for total internal reflection is

$$\alpha_{min} = i_c$$

and Snell's law becomes $$\sin \alpha_{min} = \frac{\mu_1}{\mu_2} \qquad \text{.................... (2.10-d)}$$

- In ordinary reflection 4% of the incident energy is absorbed by the interface due to refraction and absorption at every incidence but in the case of total internal reflection total incident energy is reflected back to the medium.

  This is why, using the principle of total internal reflection, optical signals are transmitted through optical fibres without any significant loss of energy. The emergent beam is as intense as the incident beam.

- In a typical optical fibre about 2 m long, a ray undergoes about 45,000 reflections. Visible light can be transmitted successfully over a length of about 50 m through a single fibre.

  For long distance transmission couplers are used to join several fibre pieces.

2.10 Basic Construction of Optical Fibres

- The transmission properties of an optical fibre depends on its structural properties. In the most widely accepted structure, an optical fibre consists of an inner solid dielectric cylinder made up of high-silica-content glass known as the ***core*** of the fibre. The core is surrounded by a solid cylindrical dielectric shell, generally made up of law-silica-content glass or plastic. This is known as ***cladding*** as shown in Fig. 2.20.

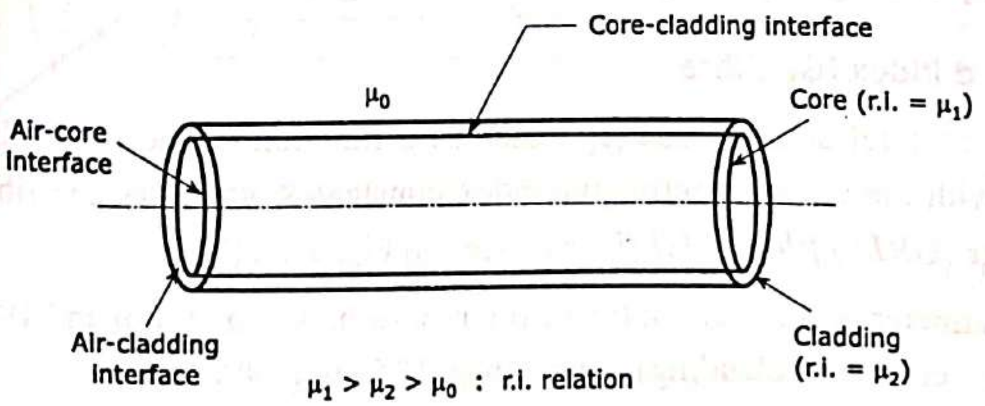
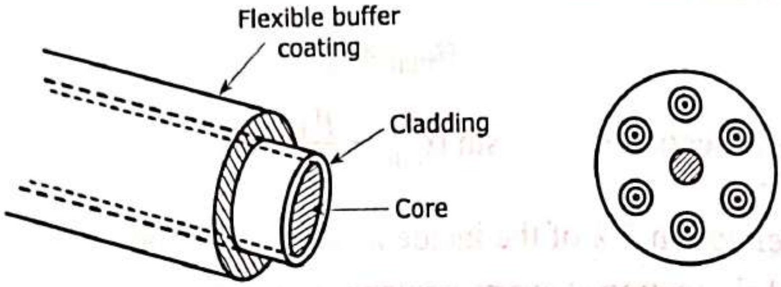


Fig. 2.20 : Structure of optical fibre

- The outermost region of an optical fibre is called the ***buffer coating***. It is a plastic coating given to the cladding for extra protection. The buffer is elastic in nature and prevents abrasions as shown in Fig. 2.21.



(a) Single optical fibre **(b)** Multi fibre cable

Fig. 2.21

- Hence, the function of the three regions of an optical fibre can be summarized as follows ;

  (i) Core : used to carry light.

  (ii) Cladding : confines the light to the core.

  (iii) Buffer coating : protects the fibre from physical damage and environmental effects.

2.10.1 : Step Index and Graded Index Fibres and their Refractive Index Profiles

Optical fibres may be classified in terms of their refractive index profiles as follows :

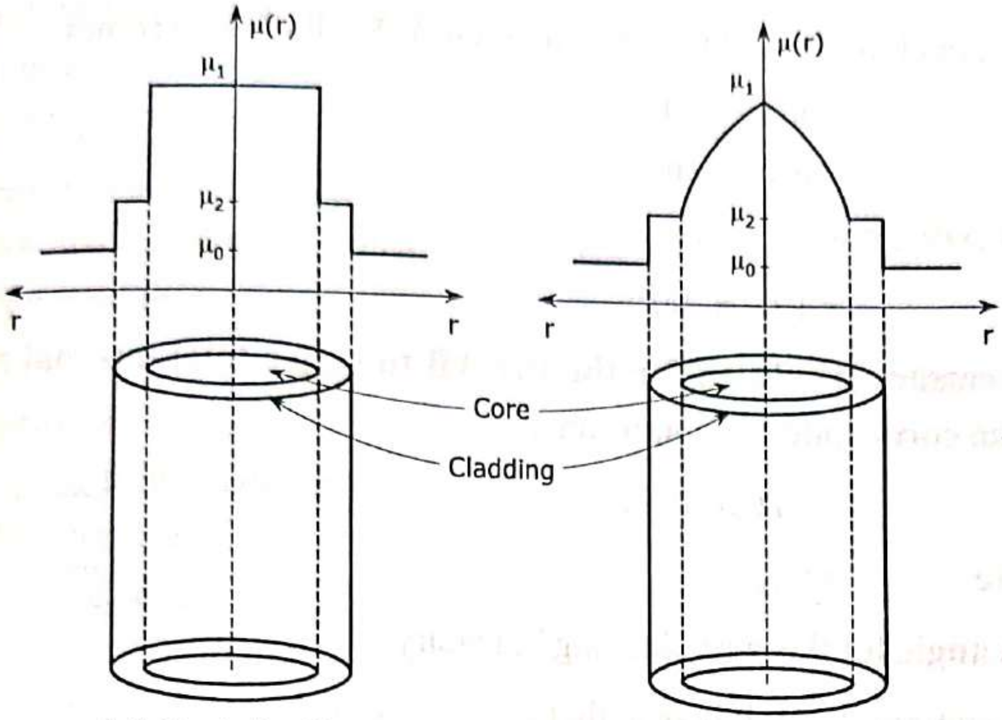
(A) Step Index (SI) Fibre

If the core refractive index remains constant at value μ_1 throughout the core region and abruptly drops to the cladding refractive index μ_2 at the core-cladding boundary it is known as ***step index fibre*** or ***SI fibre*** as shown in Fig. 2.22 (a).

(B) Graded Index (GI) Fibre

If the core refractive index μ_1 varies as a function of the radial distance 'r' as $\mu_1 = \mu_1(r)$ with the cladding refractive index constant at value μ_2, the fibre is called a ***graded index (GRIN) fibre*** or ***GI fibre*** as seen in Fig. 2.22 (b).

The diameter of a typical optical fibre ranges between 10 μm and 100 μm and the overall diameter (core + cladding) ranges from 125 μm to 400 μm.

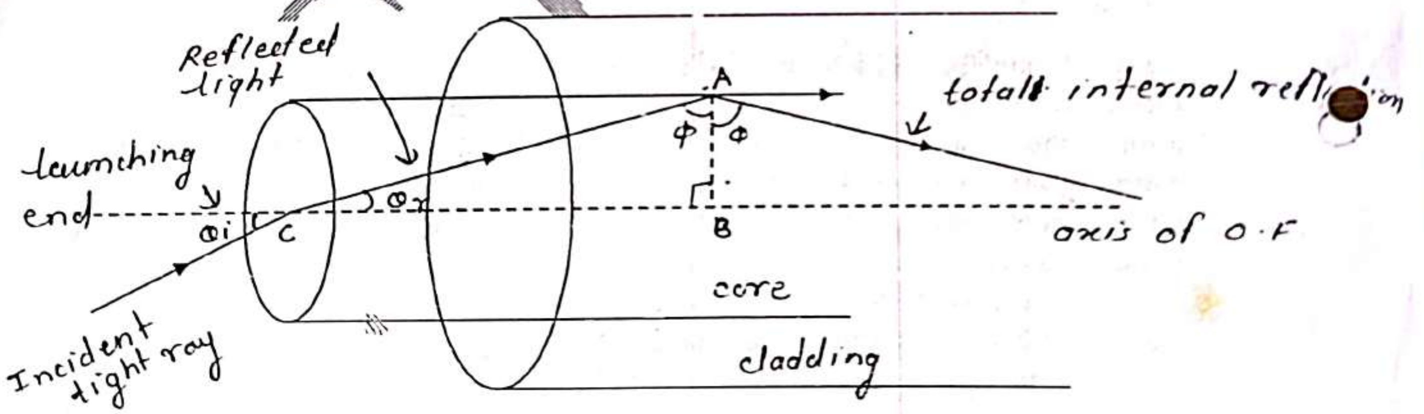


(a) Step index fibre **(b)** Graded index fibre

Fig. 2.22 : Refractive index profile

Acceptance angle

Consider a step index optical fibre into which light is launched at one end, as shown in Fig. Let the refractive index of the core be n_1 and the refractive index of the cladding be n_2 ($n_2 < n_1$). Let n_0 be the refractive index of the medium from which light is launched into the fibre. Assume that a light ray enters the fibre at an angle 'i' to the axis of the fibre. The ray refracts at an angle 'r' and strikes the core-cladding interface at an angle ϕ. If ϕ is greater than critical angle ϕ_c the ray undergoes total internal reflection at the interface, since $n_1 > n_2$. As long as the angle ϕ is greater than ϕ_c, the light will stay within the fibre.



Applying Snell's law to the launching face of fiber, we get

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{n_1}{n_0} \qquad \text{....... (1)}$$

Where n_0 is the R.I. of air.

If i is increased beyond a limit i.e. i_{max}, ϕ will drop below the critical value ϕ_c and the ray escapes from the sidewalls of the fibre. The largest value of i occurs when $\phi = \phi_c$. From the ΔABC, it is seen that

$$\sin r = \sin(90^\circ - \phi) = \cos(\phi) \quad \ldots\ldots (2)$$

Using equation (2) into equation (1), we obtain

$$\sin i = \frac{n_1}{n_0}\cos\phi \quad \ldots\ldots (3)$$

When $\phi = \phi_c$ (critical angle), for $i = i_{max}$

$$\sin i_{max} = \frac{n_1}{n_0}\cos\phi_c \quad \ldots\ldots (4)$$

But

$$\sin\phi_c = \frac{n_2}{n_1}$$

$$\cos\phi_c = \sqrt{1 - \sin^2\phi_c} = \sqrt{1 - \frac{n_2^2}{n_1^2}}$$

$$\cos\phi_c = \frac{\sqrt{n_1^2 - n_2^2}}{n_1} \quad \ldots\ldots (5)$$

From equation (4) & (5), we get

$$\sin i_{max} = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \quad \ldots\ldots (6)$$

The incident ray is launched from air medium, for which $n_0 = 1$.

$$\sin i_{max} = \sqrt{n_1^2 - n_2^2}$$

$$i_{max} = \sin^{-1}\sqrt{n_1^2 - n_2^2} \quad \ldots\ldots (7)$$

The angle i_{max} is called the **acceptance angle** of the fibre.

Acceptance angle is the maximum angle that a light ray can have relative to the axis of the fibre and propagate down the fibre.

In three dimensions, the light rays contained within the cone having a full angle $2i_{max}$ are accepted and transmitted along the fibre. Therefore, the cone is called the **acceptance cone.**

Light incident at an angle beyond i_{max} refracts through the cladding and the corresponding optical energy is lost. It is obvious that the larger the diameter of the core, the larger the acceptance angle.

Fractional refractive index change

The fractional difference Δ between the refractive indices of the core and the cladding is known as ***fractional refractive index change.*** It is expressed as

$$\Delta = \frac{n_1 - n_2}{n_1} \quad \ldots\ldots (8)$$

This parameter is always positive because n_1 must be larger than n_2 for the total internal reflection condition. In order to guide light rays effectively through a fibre, $\Delta <<< 1$. typically, Δ is of the order of 0.01.

Numerical aperture (N.A):

The main function of an optical fibre is to accept and transmit as much light from the source as possible. The light gathering ability of a fibre depends on two factors, namely core size and the numerical aperture. The acceptance angle and the fractional refractive index change determine the numerical aperture of fibre.

The numerical aperture (NA) is defined as the sine of the acceptance angle. Thus

$$NA = \sin\theta_{max}$$

$$NA = \sqrt{n_1^2 - n_2^2}$$

$$n_1^2 - n_2^2 = (n_1 + n_2)(n_1 - n_2)$$

$$= \left(\frac{n_1 + n_2}{2}\right)\left(\frac{n_1 - n_2}{n_1}\right)(2n_1)$$

Approximating $\frac{n_1+n_2}{2} \cong n_1$ we can express the above relation as

$$n_1^2 - n_2^2 = 2n_1^2\Delta$$

It gives $$NA = n_1\sqrt{2\Delta} \quad \text{.........} (9)$$

Numerical aperture determines the light gathering ability of the fibre. It is a measure of the amount of light that can be accepted by a fibre. It is seen from eq^n (9) that NA is dependent only on the refractive indices of the core and cladding materials and does not depend on the physical dimensions of the fibre. The value of NA ranges from 0.13 to 0.50. A large NA implies that a fibre will accept large amount of light from the source.

Types of optical fiber

	Step index fiber		Graded index fiber
1	Refractive index of the core- cladding is	1	Refractive index of the core is not

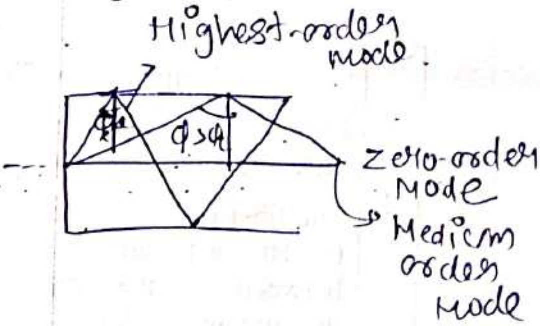
	uniform.		uniform. But, the refractive index of the cladding is uniform.
2	Since there is an abrupt change in the refractive index at the core and cladding interface, the refractive index profile takes the shape of a step. Hence, called step index fiber.	2	In this fiber, the refractive index of the core is maximum at the centre and decreases gradually (parabolic manner) with distance towards the outer edge. Hence, called graded index fiber.
3	Pulse dispersion is more in multi- mode step index fiber.	3	Pulse dispersion is reduced by a factor of 200 in comparison to step index.
4	Attenuation is less for single mode step index fiber and more for multimode step index fiber.	4	Attenuation is less.
5	Number of modes of propagation for a multimode step index fiber is given by $M_N = \frac{V^2}{2}$	5	Number of modes of propagation for a multimode graded index fiber is given by $M_N = \frac{V^2}{4}$ Thus, the number of modes is half the number supported by a MMSI fiber.

Normalized frequency (V – Number)

An optical fibre is characterized by one more important parameter, known as V-number. Which is more generally called **normalized frequency** of the fibre. The normalized frequency is a relation among the fibre size, the refractive indices, and the wavelength. It is given by

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$$

$$V = \frac{2\pi a}{\lambda}\, n_1\sqrt{2\Delta}$$



Where *'a'* is the radius of the core

And λ is the free space wavelength.

As $n_1\sqrt{2\Delta} = N.A$

We can write

$$V = \frac{2\pi a}{\lambda}(NA) \qquad \text{....... (11)}$$

The V-number determines the number of modes that can propagate through a fibre. From above equation, the number of modes that propagate through a fibre increases with increase in the numerical aperture. However, the intermodal dispersion is proportional to the square of the N.A., and therefore, more modes imply more dispersion.

The maximum number of modes M_N supported by a multimode SI fibre is given by

$$M_N = \frac{V^2}{2}$$

While the number of modes in a GRIN fibre is about half that in a similar step-index fibre,

Given by $M_N = \frac{V^2}{4}$

The axial modes are called zero order modes. The modes that propagate with $\phi = \phi_c$ are highest order modes.

The modes with $\phi > \phi_c$ are medium order modes.

2.13.1 : Fibre Optic Communication System

- Communication may be defined as the transfer of information from one place to another. For this a communication system is necessary.
- Within a communication system the information signal is superimposed on a carrier wave and the carrier wave is modulated by the information signal. The modulated carrier wave is then transmitted through the communication channel to the destination where it is received and demodulated to extract the original information.

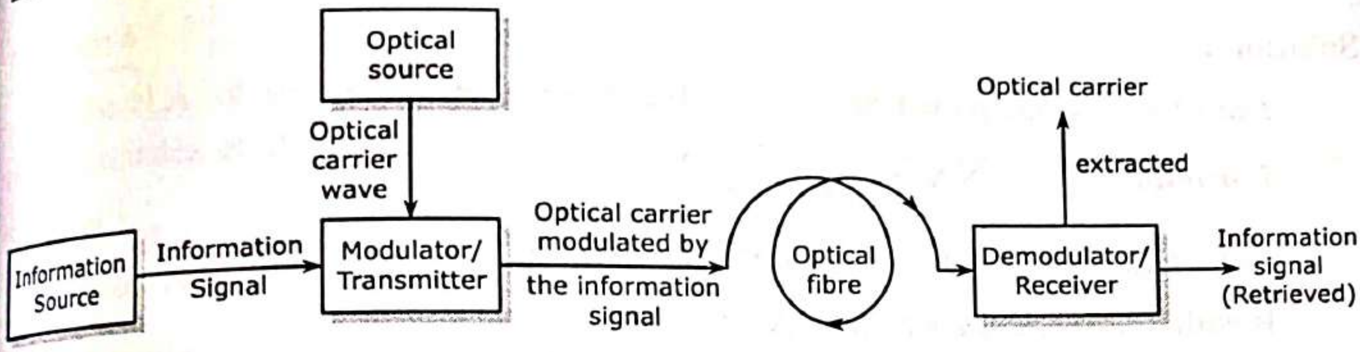


Fig. 2.29 : Optical fibre communication system

- The carrier waves are electromagnetic waves . Earlier there has been a frequent use of either the radio waves (frequency ~ 3 kHz to 300 GHz), the microwaves (frequency ~ 3 GHz to 30 GHz) or the millimeter waves (frequency ~ 30 GHz to 300 GHz), as a carrier wave.
- It has been found theoretically that the greater the carrier frequency, the larger is the transmission bandwidth and thus the information carrying capacity of the communication system.
- After the advent of laser in 1960, communication has become possible with an electromagnetic carrier selected from the optical range of frequencies.
- At higher optical frequencies (~ 10^{15} Hz) a large frequency bandwidth (~ 10^4 times the bandwidth available with a microwave carrier signal) and a high information carrying capacity (~ 10^5 times the information carrying capacity of a microwave carrier signal) are available.
- However, light energy gets dissipated in open atmosphere by inverse square law,

$$I \propto \frac{1}{d^2}$$

where 'I' is the intensity of the light beam and 'd' is the distance travelled.

This dissipation is caused by the diffraction and scattering of light by dust particles, water vapour etc. and due to absorption in the medium.

- Hence, to transmit an optical carrier signal over a long distance a guiding channel is required. This is done by sending an optical beam or pulse through an optical fibre.

5.3 Nanomaterials

The materials with structural units which are an aggregate of atoms or molecules with dimensions in Nano scale *i.e.*, between 1 nm and 100 nm are called *nanomaterial.* Engineered nanomaterials are produced with required dimensions *i.e.*, either one or two or all the three dimensions in the nanoscale.

- Nano materials that have at least one dimension in the nano scale are called ***nanolayers***, such as thin films or surface coatings.
- If two dimensions of a nanomaterial are in the nano scale they are categorized as ***nanotubes or nanowires.***
- Lastly, nanomaterials that have all the three dimensions in the nano scale are called ***nanoparticles.***
- Nanomaterials made up of nanometer sized grains are called ***nanocrystalline solids.***

5.3.1 : Properties of Nano materials

The properties of nano materials are very different from those of the bulk materials. One important difference is the increased surface area to volume ratio of nanostructures. Nonstructures are also associated with quantum effects. These special properties are due to the size of the nano particles.

(a) Optical Properties

Depending upon their constituents, nanoparticles absorb a range of wavelengths and emit a characteristic wavelength. It is possible to alter the linear and non-linear optical properties by altering the crystals. Nonomaterials are, therefore, used in electrochromic devices.

When light is incident on a nanoparticle it can be scattered or absorbed. The total effect of scattering and absorption is referred to as extinction. Nanoparticles are in the size regime where the fraction of light that is scattered or absorbed can vary greatly depending on the particle diameter. At diameters less than 20 nm, nearly all of the extinction is due to absorption. At sizes above 100 nm, the extinction is mostly due to scattering. By designing a nanoparticle with desirable diameter the optimal amount of scattering and absorption can be achieved.

(b) Electrical Properties

The size of nanomaterials leads to an increase in their ionization potential. Due to quantum confinement the electronic bands come closer and become narrow. Energy states are transformed into localized molecular bonds which can be altered by the passage of current or by the application of a field. The change in electrical properties is material dependant. As an example, metals undergo an increase in conductivity whereas in the case of non-metallic nanomaterials a decrease in conductivity is observed.

(c) Magnetic Properties

Nanosized materials are more magnetic than their counterparts in the bulk. Nanoparticles of non-magnetic solids also may demonstrate magnetic properties.

The dynamics of magnetization and demangetization of magnetic materials in any device are governed by the presence of domain walls and regions with magnetization in different directions. In the case of magnetic nanoparticles, the magnetic vectors become aligned in the ordered pattern of a single domain in the presence of a DC magnetic field. In such cases, phenomena of thermal excitation or quantum mechanical tunnelling change the hysteresis loop of magnetic nanoparticles as compared to the bulk material.

(d) Mechanical and Structural Properties

Due to the formation of nanoparticles the atoms which are on the surface face different potentials in different directions. The resulting surface stress in nanoparticles modifies its mechanical and structural properties. The intrinsic elastic modulus of a nanostructured material is essentially the same as that of the bulk material having the micrometer sized grains until the grain size becomes very small, < 5 nm. If the grain size is below 20 nm the Young's modulus of the material begins to decrease from its value in conventional grain sized materials. Most nanostructured materials are quite brittle and display reduced ductility under tension.

In nanomaterials, because of their nanosize many of their mechanical properties are modified from its value in bulk materials. These properties among other are hardness and elastic modulus, fracture toughness, scratch resistance and fatigue strength. Energy dissipation, mechanical coupling and mechanical non-linearities are influenced by structuring components at the nanometer scale.

5.3.2 : Surface Area to Volume Ratio

The surface area to volume ratio determines the efficiency of the object. The surface area to volume ratio for a material or substance made of nanoparticles has a significant effect on the properties of the material. Nanomaterials have much greater surface area per unit volume ratio compared with the bulk materials.

Take for example, a cube with side length 'a'.

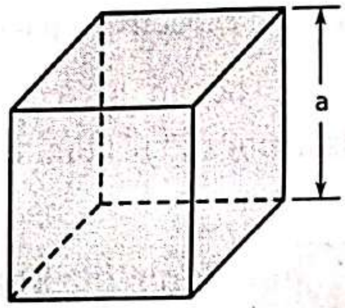


Fig. 5.3 (a)

The surface area of the cube is

$$S = 6a^2$$

The volume of the cube is

$$V = a^3$$

The surface to volume ratio is given by

$$\frac{S}{V} = \frac{6}{a}$$

If a = 2 cm, $\frac{S}{V} = 3\ \text{cm}^{-1}$

Now, lets consider a sphere of radius 'a'.

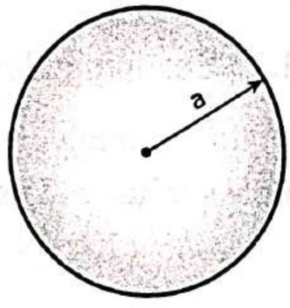


Fig. 5.3 (b)

The surface area is $S = 4\pi a^2$.

The volume is $V = \frac{4}{3}\pi a^3$

The surface to volume ratio is given by

$$\frac{S}{V} = \frac{3}{a}$$

If a = 2 cm, $\frac{S}{V} = 1.5\ \text{cm}^{-1}$

The cube with larger S / V ratio than that of sphere is considered as more efficient in nanotechnology. ***The more the S / V ratio, the greater is the efficiency of the nanomaterial.***

5.3.3 : Two Main Approaches in Nanotechnology

The two approaches used in nanotechnology to prepare nanomaterials are top down approach and bottom up approach which are explained below.

(i) Top Down Approach

In this technique nanostructures are fabricated by reducing a bulk material to nanoparticles through methods as cutting, carving and moulding. Despite the fact that those techniques introduce various structural defects in the material it is widely used in nanotechnology due to its simplicity.

(ii) Bottom Up Approach

In this technique, nanostructures are built up atom by atom or molecule by molecule. Even the nanostructure formed by a single molecule can be developed. The information storage capacity of nanostructures constructed in this approach is very high.

Though this technique does not cause much damage to the structure of the material its application is limited due to the complexities involved.

5.4 Tools for Characterization of Nanoparticles

Several forms of microscopy are available for studying nanomaterials are discussed below. Three most commonly used microscopies are as follows :

5.4.1 : Scanning Electron Microscope (SEM)

In scanning electron microscope an electron beam is made to be incident on the sample surface and its image is formed by the emitted secondary electrons, back scattered electrons and X-rays.

Principles

It is based on the wave nature of electrons and the interactions of high energy electrons with the sample surface.

Construction

A schematic diagram of SEM as shown in Fig. 5.4.

- There is an electron gun comprising of a filament and a cathode which emit a beam of thermionically emitted electrons.
- The electron beam passes through two pairs of condenser lenses C_1 and C_2.
- The condensed electron beam then passes through a scanning coil S.
- Before being incident on the sample the electron beam passes through the objective lens.

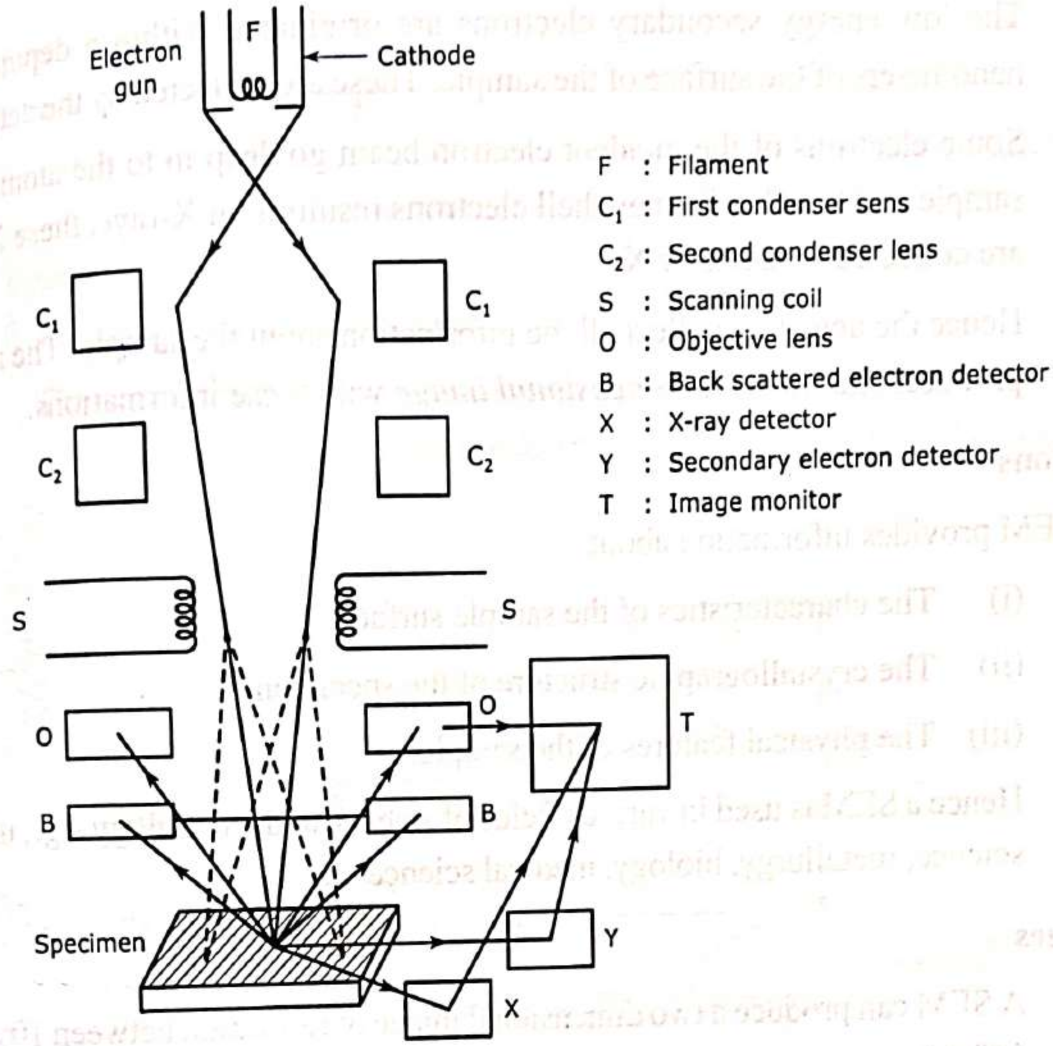


Fig. 5.4 : Scanning Electron Microscope (SEM)

- The detectors are used to detect the back scattered electrons, the secondary electrons and the X-rays.
- Taking input from the detectors the image is produced on the monitor.

orking

- The electron gun produces a high energetic electron beam.
- The condensed lenses focus the diverging electron beam into a fine beam of a spot diameter of few nanometers.
- The scan coils deflect the electron beam in various directions to scan across the surface of the sample.
- The objective lens is used to focus the beam at a particular point on the sample surface.
- The back scattered electrons are reflected from the surface of the sample and are collected by the detector, B.

- The low energy secondary electrons are originated within a depth of few nanometers of the surface of the sample. These are collected by the detector Y.
- Some electrons of the incident electron beam go deep in to the atoms of the sample and knock off inner shell electrons resulting in X-rays, these X - rays are collected by detector X.
- Hence the detectors collect all the information about the sample. The monitor produces the final ***two dimensional image*** with these informations.

Applications

A SEM provides information about

(i) The characteristics of the sample surface.

(ii) The crystallographic structure of the specimen.

(iii) The physical features of the sample.

- Hence a SEM is used in various fields of science and technology *e.g.*, material science, metallurgy, biology, medical science etc.

Advantages

- A SEM can produce a two dimensional image of resolution between 10 A° and 100 A°.
- A SEM has a very high magnifying power.

Disadvantage

- A SEM can produce an image of the surface of the sample and not of its interior.
- The sample to be studied with a SEM is required to be conducting. For no conducting samples a thin conducting coating on the top surface is used.

5.4.3 : Atomic Force Microscope (AFM)

The atomic force microscope is a scanning probe microscope used as an imaging device.

Principle

The various types of forces experienced by the probe while scanning the sample surface scatters a LASER signal which is turn produces a three dimensional image of the sample.

Construction

- The AFM consists of a probe with a sharp tip fitted to a cantilever. The radius of the tip is around 1 nm and the length of the cantilever is around 10 nm as shown in Fig. 5.6.

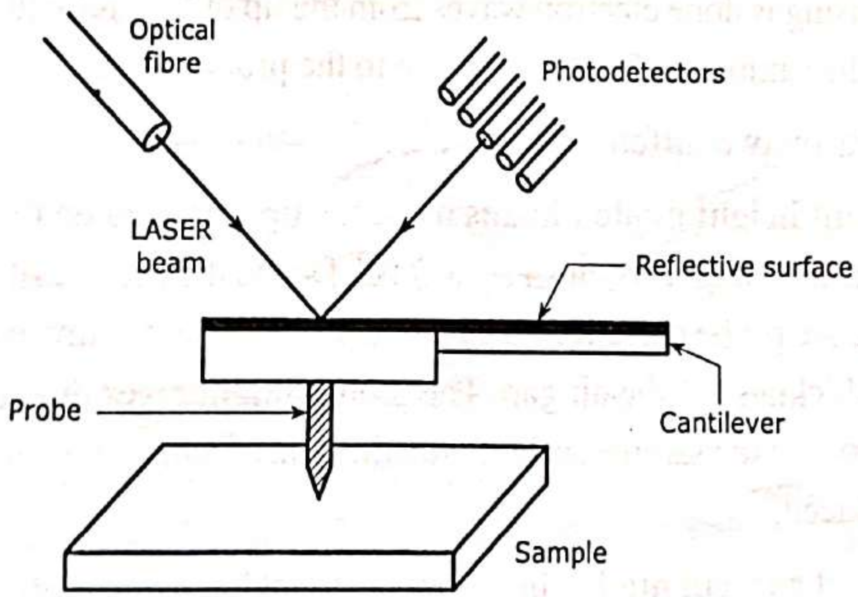


Fig. 5.6 : Atomic Force Microscope

- The cantilever surface is highly reflective. From a LASER source a laser beam and made to be incident on the cantilever through an optical fiber. The reflected LASER beam is collected by a series of photo detectors.

Vorking

- As the probe is moved over the sample surface the tip experiences a force due to which the cantilever undergoes a deflection.
- According to the type of the sample the force can be of electrostatic, magnetic, mechanical and even van Der Waals forces.
- The interactive force is detected by a series of photo detectors which collect the LASER beam scattered at different direction due to the deflection of the probe.
- The three dimensional image carrying the information of the topography of the surface is then formed.

ιdvantages

- The resolution of the image is in nanometer range.
- Both the conducting and non conducting surfaces can be scanned by an AFM.

)isadvantage

The scanning process is slow.

ιpplications

AFM can be used to study various types of samples, *e.g.*, conductors, semiconductors, ısulators biological tissues etc. AFM is also used to form nanoparticles with its fine probe.

.4.4 : Comparison of SEM and AFM

Table 5.1

Sr. No.	SEM	AFM
1.	The sample needs to be conducting.	The sample can be conducting or nonconducting.
2.	The operation requires vacuum.	The operation is possible in open atmosphere.
3.	Resolution of the image is more.	Resolution of the image is less.
4.	It produces a two dimensional image.	It produces a three dimensional image.

5.5 Methods to Synthesize

Various methods used for the production of nanomaterials are described here.

(a) Mechanical Method : Ball Milling Method

In this method small hard steel balls are kept in a container filled with the powder of the bulk material. The container spins about itself while rotating in a circular path about a central axis like a planet moves around the sun. The size of the steel balls used in milling is inversely proportional to the size of the nanoparticles they produce. This is a simple, economical method that can be used at room temperature. This is used to make nanoparticles of metals and alloys.

(b) RF Plasma Technique : Sputtering

- In this technique the bulk material is kept in a pestle which is kept in an evacuated chamber as shown in Fig. 5.7.

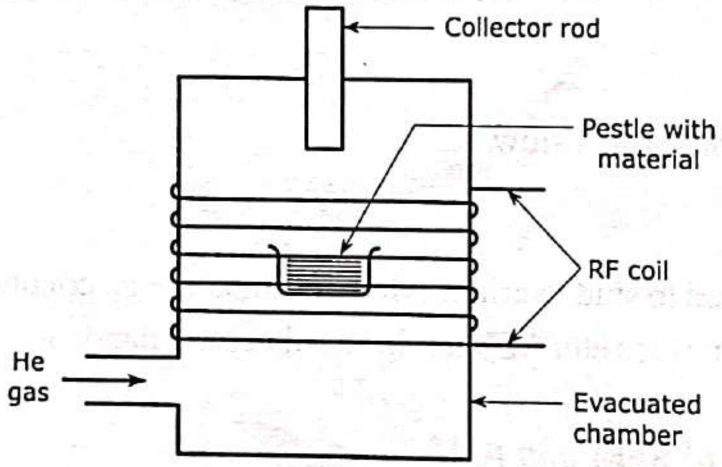


Fig. 5.7 : RF Plasma

- When a high voltage is applied to the RF coils heat in generated and the evaporation of the metal begins. Then cold He gas is allowed to enter the chamber. This results in high temperature plasma in the region of the coils. Nanoparticles are formed from the metal vapor and are collected by the collector.

(c) Inert Gas Condensation : Vapour Diposition

This is the primitive technique of synthesizing nanomaterials. In this technique a metallic or inorganic material is vaporized. In the evaporation process ultrafine particles are formed. These particles rapidly form clusters which in turn condense into crystallites. Using a rotating cylinder and a cold finger both maintained at liquid nitrogen temperature ne nanoparticles are removed from the gas. This method is very useful to produce composite naterials.

d) **Chemical Solution Deposition Method : Sol-Gel Method**

A sol is a solution with particles suspended in it. When the particles in the sol form ong polymers throughout the sol it becomes a gel. The sol-gel process is a bottom up pproach technique. The bulk material is converted to a powder and mixed in a chemical olution to form the sol. The sol is then partly converted to gel. The sol-gel solution through avitation effect produce the nanoparticles.

Sol-gel synthesis is superior of all the available processes as it can produce large uantities of nanomaterials at relatively low cost. In this technique almost any material an be synthesized. It is very useful in producing extremely homogeneous alloys and omposites controlling the physical, chemical and mechanical properties and the nicrostructure of the developed nanostructure.

e) **Laser Ablation**

In this method a very high intensity (> 10^7 w/cm^2 pulsed laser beam is focused on he material target. The pulsed laser generates very high temperature (> 10^4 K) at the target lement resulting in the vaporization of the material. A cool, high-density helium gas is nade to flow over the target resulting in the formation of clusters of the target material. The clustered material is then thermalizes to room temperature and finally cooled to a few K to produce nanomaterials.

This technique has an extensive use because of the fact that a wide range of bulk naterial can be used in this top-down kind of approach.

f) **Thermolysis**

In this process the nanoparticles are formed by decomposing solids at high temperature naving metal cations and molecular anions or metal organic compounds.

5.6 Applications of Nanomaterials

Nano materials have a wide verity of applications some of which are explained below.

1. **Self cleaning glass :** Nanoparticles are coated on a glass surface to make it photocatalytic and hydrophilic. In photocatalytic effect when UV radiation falls on the glass surface, the nanoparticles become energised and begin to

break down the organic particles on the glass surface. On the otherhand, due to the hydrophilic nature the glass attracts water particles which then clean it.

2. **Clothing :** Clothing with improved UV protection are manufactured by applying a thin layer of zinc oxide nanoparticle on it.

   Also clothes can have nanowhiskers that can make them repel water and other materials thus making them stain resistance.

   Silver nanoparticles coating can have an antibacterial effect on the clothes.

3. **Scratch resistant coating :** Materials like glass are being coated with thin films of hard transparent material to make it scratch resistant.

   Antifog glasses with transparent nanostructures conduct electricity and heat up the glass surface to keep it fog free.

4. **Smart materials :** Nanotechnology enabled smart materials may be able to change and recombine much like the shape shifting cyborg in the movie terminator 2. They may incorporate nonsensors, nanocomputers and nanomachines into their structure which may enable them to respond directly to their environment.

5. **Cutting Tools :** Cutting tools made of nanocrystalline materials are much harder, much wear-resistant and very long lasting.

6. **Insulation Materials :** Nanocrystalline materials synthesized by the sol-gel techniques results in a foam like structure called aerogel. These are porous and extremely light but can withstand heavy weight. These are very good insulators. Aerogels are also used to boost the efficiency of transducers.

7. **Ductile, Machinable ceramics :** Normal ceramics are very hard, brittle and difficult to machine. However, the nanocrystalline ceramics possess good formability, good machinability combined with excellent physical, chemical and mechanical properties.

8. **Low-cost flat-panel electrochromic displays :**

   - Electrochromic devices are very similar to liquid crystal displays. These devices display information by changing color when a voltage is applied. If the polarity of the voltage is reversed the colors gets bleached.
   - If nanocrystalline materials are used in these devices the resolution, the brightness and the contrast of the display increases greatly.

Nanosized titanium dioxde and Zinc oxide are used in some sunscreens, as they absorb & reflect UV rays.

9. **Elimination of pollutants :** Since nanomaterials exhibit enhanced chemical activity they can be used as catalysts to react with pollutants like carbon monoxide and nitrogen oxide to prevent environmental pollution arising from burning gasoline and coal.

10. **High power magnets :** The nanocrystalline magnets have very high magnetic strength given by its coercivity and saturation magnetization value. These magnets have applications in automobile engineering, marine engineering, in medical instruments like MRI etc.

11. **High energy - density batteries :** Nanocrystalline materials synthesized by sol-gel treatment has foam like structure which can store a large amount of energy hence batteries with separator plates made up of these materials do not need frequent changing.

12. **High sensitivity sensors :** Sensors made of nanocrystalline materials are extremely sensitive to the change in their environment. These sensors are used as smoke detectors, ice detectors on aircraft wings, automobile engine performance sensor etc.

13. **Aerospace components :** Aerospace components made of nanomaterials are stronger, tougher and more long lasting than those with conventional materials. This increases the life of the aircraft greatly.

Important Points to Remember

The surface area to volume ratio : This determines the efficiency of the object.

Two approaches : Top down and bottom up.

SEM : Electron wavelength : $\lambda = \frac{h}{\sqrt{2meV}}$.

Sample needs to be conducting.

Operation are possible only in vacuum.

A high resolution two dimensional image is formed.

STM works on quantum mechanical tunneling effect.

Sample should be conducting.

A three dimension contour of the sample surface is imaged at atomic scale.

Nanosized iron oxide is present in some Lipsticks as a pigment.

Laser & fibre optics:

Introduction:-

The term LASER stand for light Amplification by stimulated Emission of Radiation.

The first succesful operation of laser was demonstrated by T. Maimon in 1960 using a ruby crystal in USA.

→ Since laser is high energy beam, at times it is compared with X-Rays. But both of them differ completely. Some points at which LASER differs from X-Rays are

(a) Laser is highly coherent where an X-Ray is not.
(b) Laser has its wavelength of the order of visible spectrum where as X-Rays has very small wavelength.
(c) stimulated Emission is essential for LASER where as X-Ray needs high energy electron and their retardation.

Charecteristics of LASER light-

→ like ordinary light, laser light is electromagnetic in nature. However there are few charecteristics of laser light not possessed by normal light.
Some of the charecteristics of laser lights are mentioned below.

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(1) Directionality :-

The laser beam is highly directional having almost no divergence.
The output beam of a laser has a well-defined wavefront and therefore, it is highly directional.
The normal light from a strong source spreads to about one kilometer for every kilometer of its propogation. Just imagine how much a normal beam of light would diverge when it reaches moon at a distance of 3,80,000 km while a laser beam spreads to just a few kilometer on reaching the moon.

(2) Monochromaticity :-

The laser light is nearly monochromatic. In reality, no light is perfectly monochromatic i.e. it is not characterized by a single wavelength (or frequency) but instead it is characterized by spread in frequency $\Delta\nu$ about the central frequency.
The monochromaticity of light is defined by $\Delta\nu/\nu$. For perfectly monochromatic $\Delta\nu = 0$ which is not attainable in practice, but the value of $\Delta\nu$ is much smaller for lasers compared to ordinary light.

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(3) Coherence :-

Laser radiation is characterized by high degree of coherence, both spatial and temporal.
In other words, a constant phase relationship exists in the radiation field of laser light source at different location and time.

(4) Intensity :-

The laser beam is highly intense compared to ordinary light. Since, the laser power is concentrated in a beam of very small diameter (few mm); even a small laser can deliver very high intensity at the focal plane of the lens.
Note that even a small power of 1 watt can give an intensity of $10^9\ W/m^2$, which is extremely high.

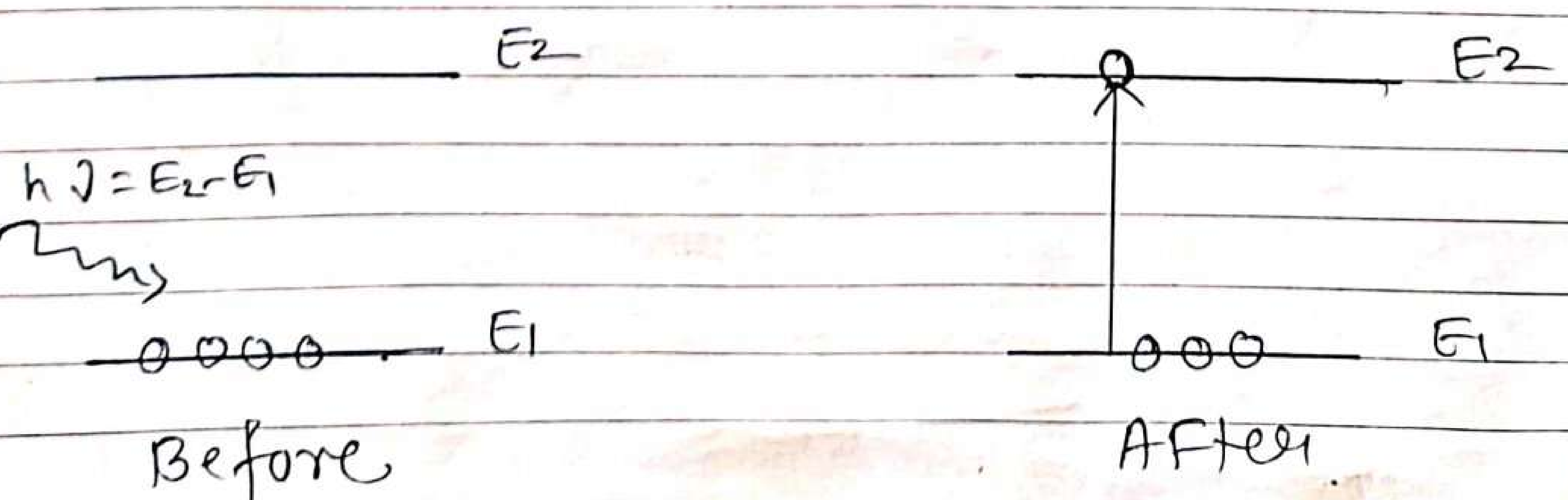
To understand the principle of working of laser, it is necessary to understand the relative absorption and emission of photons in atomic system. According to quantum theory, atoms exists only in certain discrete energy states and absorption and emission of photons causes them to make a transition from one discrete state to another.

Transition from one energy level to another can occur by stimulated absorption (or simply absorption) spontaneous emission and stimulated emission.

Absorption :-

(1) Following fig shows the absorption process in a two ~~level~~ energy level atom.

(2) The atoms are initially in the lower energy state E_1



(3) when a photon is incident on the atom the atom absorbs a photon of frequency ν such that $h\nu = E_2 - E_1$ and makes a transition to higher energy level E_2.

(4) This process is known as absorption. The process is represented as

Atom + Photon = Atom in excited state.

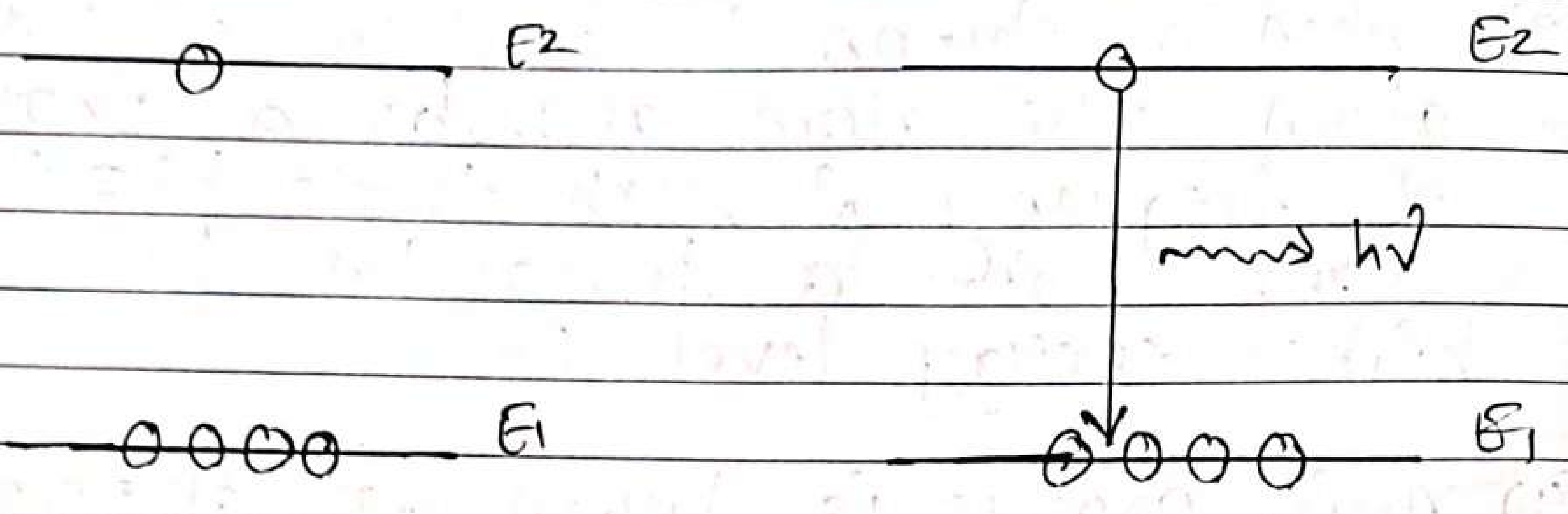
(5) The Rate of absorption is directly proportional to number of atoms present in lower energy state and energy density of photons

$$\therefore \; R_{ab} \propto N_1 \, u(\nu)$$
$$\therefore \; R_{ab} = B_{12} N_1 \, u(\nu)$$

where B_{12} is known as Einstein's coefficient of absorption.

Spontaneous Emission

(i) when an atom undergoes transition to a lower energy state emitting a photon without any external stimulation, the process is termed as spontaneous emission as shown in fig.



(2) The process is represented as

Atom in Excited state = Atom + Photon

(3) The photons emitted by different atoms by Spontaneous emission are incoherent and non-monochromatic.

(4) light from an ordinary light source including LED is given out by this process and hence is incoherent.

(5) The Rate of Spontaneous emission is directly proportional to number of atoms present in higher energy states.

$$R_{sp} \propto N_2$$

$$R_{sp} = A_{21} N_2$$

where A_{21} = Einstein's coefficient of Spontaneous emission

Stimulated Emission :-

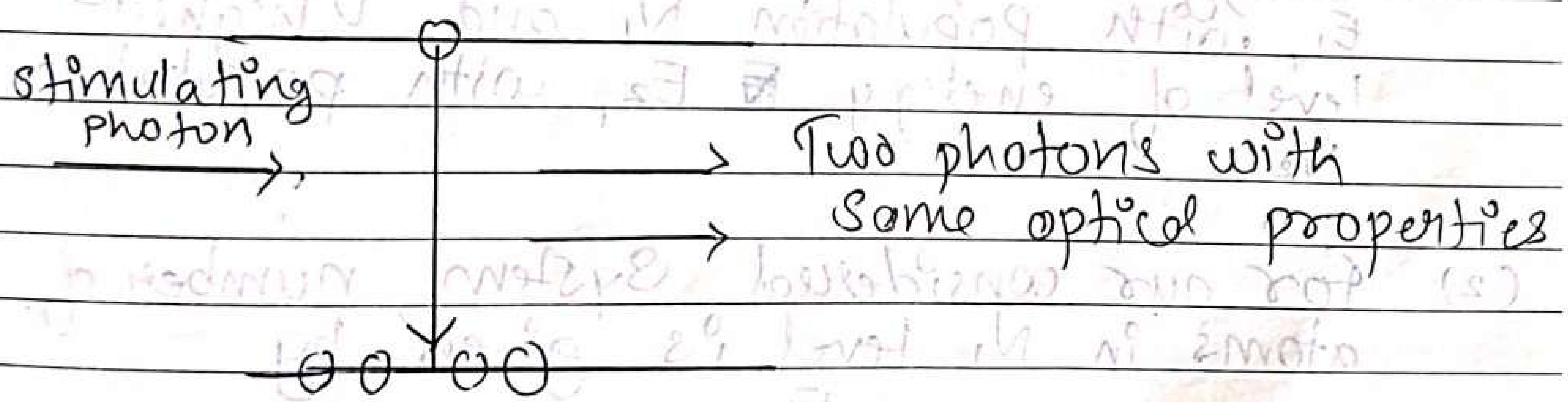
(1) An atom in excited state need not wait for spontaneous emission to occur. There exist an additional possibility according to which an excited atom can have a downword transition and emit a radiation.

(2) A photon of energy $h\nu = E_2 - E_1$ can induce the excited atom to make a downword transition releasing the energy in the form of photon.

(3) This phenomenon is called forced emission or stimulated emission.

The process may be represented as.

Atom in excited state + Photons = Atom + photon + Photon



(4) In this process a new photon of the same frequeny, phase, Polarization and direction of propogation as the stimulating photons is generated.

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(5) The rate of stimulated emission is proportional to Number of atoms in exited state and energy density of photons.

$$R_{ST} \propto N_2 u(\nu)$$

$$R_{st} = B_{21} N_2 u(\nu)$$

where B_{21} = Einstein's coefficient of stimulated emission

Relation between Einstein's coefficient

(1) In a process of transition, two energy levels are involved.
Let us consider a system having two energy level, a lower level of energy E_1 with population N_1 and higher level of energy E_2, with population N_2.

(2) For our considered system number of atoms in N_1 level is given by

$$N_1 = N_0 e^{-\frac{E_1}{KT}} \quad \text{——} \quad (1)$$

where N_0 = No. of atoms in ground state.
Number of atoms in N_2 level is given by

$$N_2 = N_0 e^{-\frac{E_2}{KT}} \quad \text{——} \quad (2)$$

$$\therefore \frac{N_1}{N_2} = \frac{N_0\, e^{-E_1/KT}}{N_0\, e^{-E_2/KT}}$$

$$\therefore \frac{N_1}{N_2} = e^{\frac{(E_2-E_1)}{KT}} \quad \text{——— (3)}$$

(3) In closed System, under thermal Equilibrium, the net rate of emission must equal the net rate of absorption.

$$\therefore R_{st} + R_{sp} = R_{ab}$$

$$\therefore B_{21} N_2\, u(\nu) + A_{21} N_2 = B_{12} N_1\, u(\nu)$$

$$u(\nu) = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2}$$

$$u(\nu) = \frac{\frac{A_{21} N_2}{B_{21} N_2}}{\frac{B_{12} N_1}{B_{21} N_2} - 1}$$ [Divide all terms on R.H.S by $B_{21}N_2$]

$$\therefore u(\nu) = \frac{\frac{A_{21}}{B_{21}}}{\frac{B_{12}}{B_{21}} e^{(E_2-E_1)/KT} - 1} \quad \text{——— (4)}$$

(4) The energy density $u(\nu)$ is given by Planck's radiation law as

$$u(\nu) = \frac{\frac{8\pi h\nu^3}{c^3}}{e^{(E_2-E_1)/KT} - 1} \quad \text{——— (5)}$$

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(5) comparing eqn (4) and (5) we get

$$B_{12}/B_{21} = 1$$

$$\therefore B_{12} = B_{21}$$

and $\frac{A_{21}}{B_{21}} = \frac{8\pi h \nu^3}{c^3}$ } (6)

(6) The above expressions (6) are known as Einstein's relations.

(7) These relations led us to following conclusions
(a) The probability of absorption and stimulated emission are equal.
(b) The ratio of probabilities of Spontaneous and stimulated emission is proportional to ν^3.

Population Inversion

(1) In an assembly of identical atoms in thermal equilibrium if N_1 and N_2 are the number of atoms in two state of energies E_1 and E_2 $E_1 < E_2$ then N_2 is always lesser than N_1.
(2) In fact, most of the atoms are in the ground state and the number of atoms decreases sharply as we go to higher states.
(3) It is obvious that if this system is exposed to ~~atoms~~ the radiation of frequency $\nu = E_2 - E_1/h$ than most of the photons will be absorbed and stimulated emission will be negligible.

(4) Therefore, production of laser light will not be possible.

(5) However, if we have a situation in which majority of the atoms are in the higher state E_2 than in the lower state E_1 then stimulated emission will dominate and laser action will be possible.

(6) Such a reversal of the normal population is called Population inversion.

Pumping :- Types of Pumping

Pumping :- The process of raising the atoms or molecules of the active medium to a higher energy states, so as to acheive the population inversion is called as Pumping

The Pumping methods generally used are

(a) Optical Pumping :- The Method of Pumping in which the optical energy is used is known as the optical pumping.
High intense beam from the optical flash lamp is made incident on the active Medium. Generally this Method is used in the Solid state lasers.

(b) Electrical Pumping :-

An electric discharge is used to excite the atoms of the active medium. This type of pumping is used in the gas lasers. An extremely high electric field accelerates the electrons emitted by cathode towards the anode. Collision between the high energy electrons and the atoms of the active medium raise them to the excited state to acheive the population inversion.

(c) Chemical Pumping :-

In chemical laser, the energy required for excitations is obtained from a suitable chemical reaction.

(d) Direct Pumping :-

In Semiconductor laser and LED direct conversion of electrical energy into radiation take place.

Metastable State :-

(1) An atom can be excited to a higher level by supplying energy to it. Normally excited atoms have short lifetimes and release their energy in a matter of 10^{-8} sec through spontaneous emission.

(2) Population inversion cannot be established under such circumstance. In order to establish population inversion the excited atoms are required to wait at the upper level till a large no. of atoms accumulate at that level.

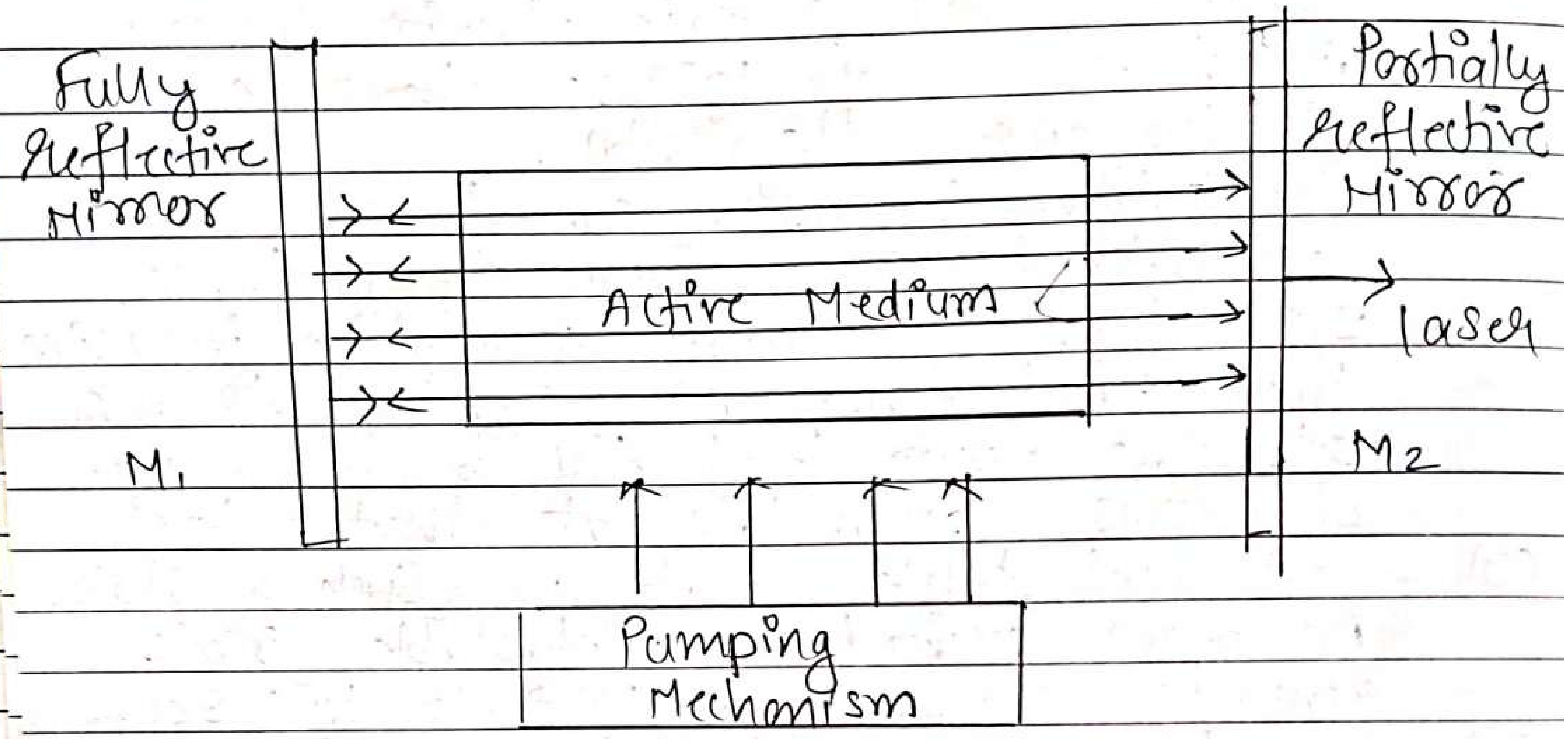
(3) A metastable state is such a state. An atom excited to metastable state remains excited for 10^{-3} to 10^{-2} sec.

(4) Therefore, the metastable state allows accumulation of a large number of excited atoms at that level.

(5) It would be impossible to create the state of population inversion without a metastable state.

Basic components of laser

A laser has three essential components as shown in fig.



(1) A material which on being exited sustains population inversion and subsequently lases. Such a material is called an Active Medium. The Medium may be solid, liquid or gas.

(2) A Pumping Mechanism that raises the active Medium to an exited state.

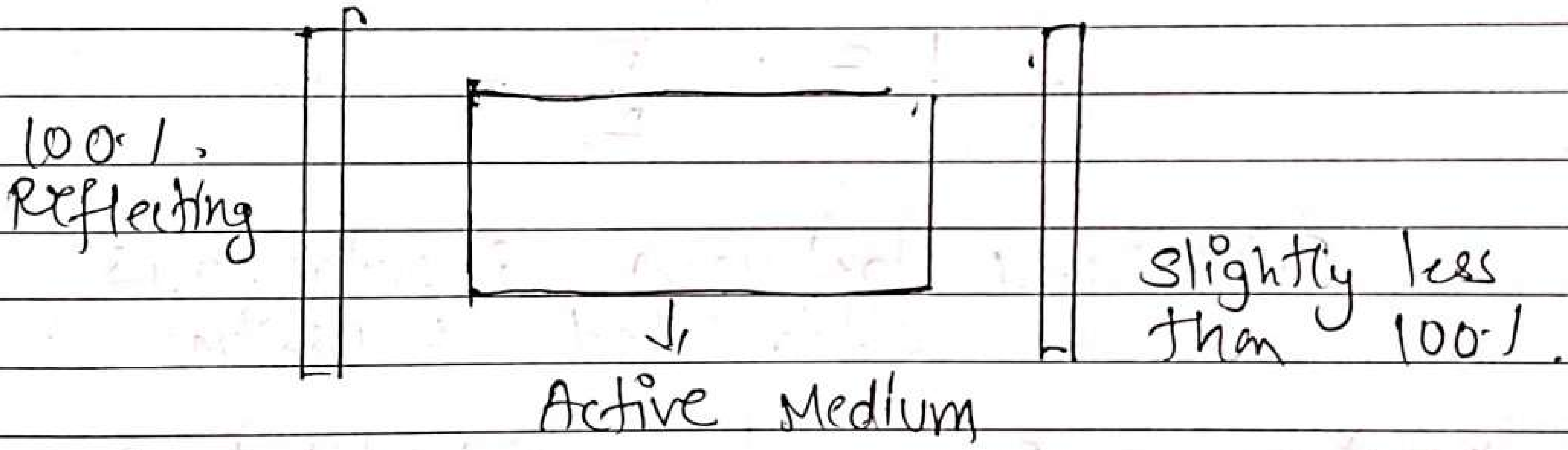
(3) The photons produced due to stimulated emission are sent back and forth through the Medium many time by Mirror M_1 and M_2 so as to stimulate further emissions. The Mirror M_1 is fully reflective mirror while M_2 is partially reflective. Therefore a small fraction of the intense beam emerges from M_2.

Resonant Cavity

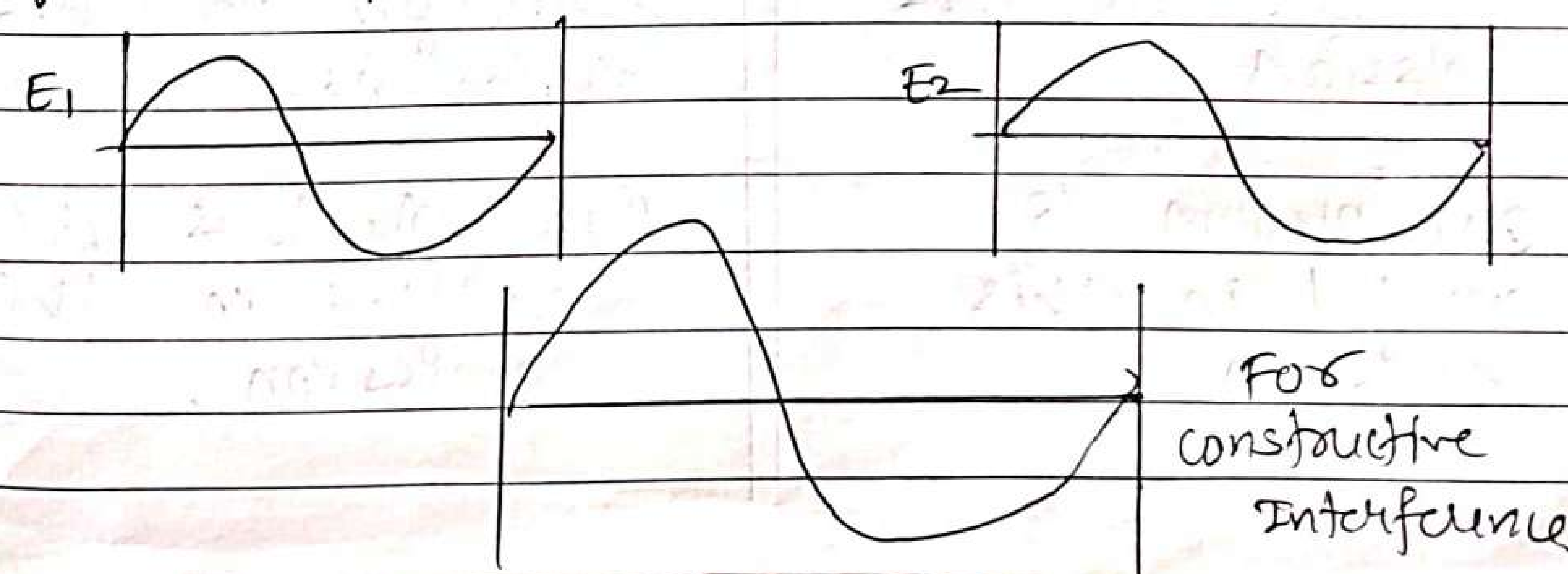
(1) A laser device consists of an active Medium bound between two mirrors. These surfaces reflect the photons to and fro through the active Medium.

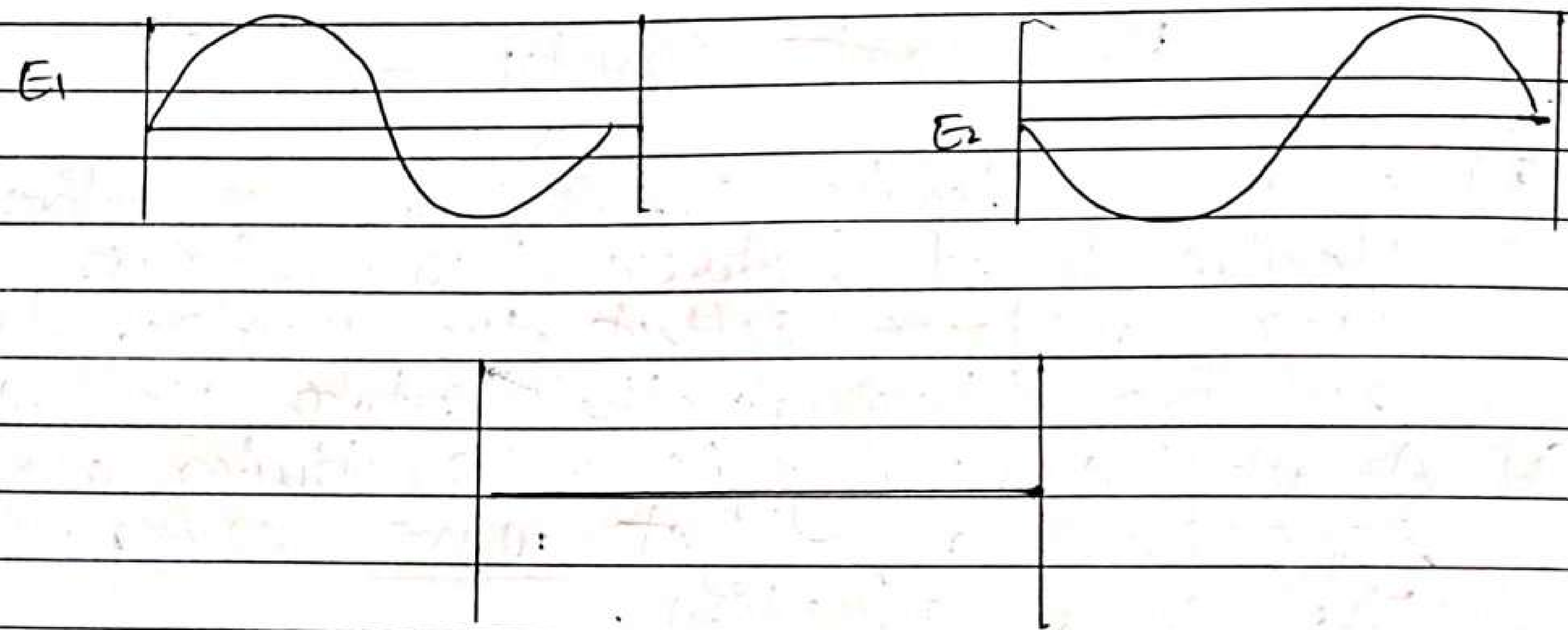
(2) A photon moving in a particular direction represents a light wave moving in the same direction.

(3) Thus the two mirrors along with the active Medium form a cavity inside which two types of waves exists one wave moving towards right and the other one to left.



(4) The two waves interfere constructively if there is no phase difference between the two waves.
But their interference becomes destructive if the phase difference is $\pi/2$.





For Destructive Interference

(5) In order to arrange for constructive Interference the distance 'L' between two reflecting surfaces should by integral Multiple of half wavelength.

$$\therefore L = m\frac{\lambda}{2}.$$

Difference between Spontaneous & Stimulated Emission:

Spontaneous Emission	Stimulated Emission
① It was postulated by Bohr	It was postulated by Einstein
② Additional photons are not required in this emission	Additional photons are required in this emissions.
③ one photon is emitted in this emission	Two photons are emitted in this emission.

(4) The emitted radiation is ~~inh~~ incoherent	The emitted radiation is coherent
(5) The emitted radiation is polymonochromatic	The emitted radiation is Monochromatic
(6) The emitted radiation is less intense	The emitted radiation is high intense.

required.

The Helium-Neon Laser:

The helium-neon laser is the most commonly used gas laser. It was first developed by Ali Javan and his collaborators in 1961. It is a **four-level laser** and provides a continuous supply of laser beam.
The schematic diagram of the essential components of a He-Ne laser is shown in Fig.7.

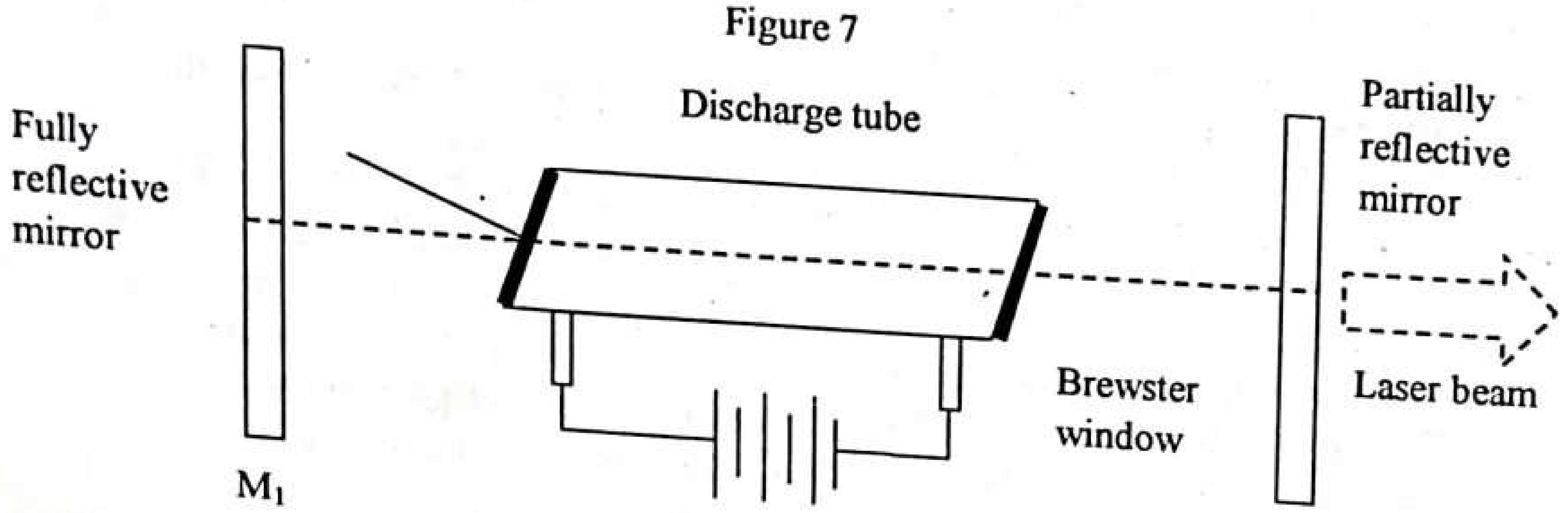
Construction:

1. He-Ne laser consists of a long (10 to 100 cm) and narrow (diameter 2 to 10 mm) discharge tube filled with helium and neon gases with typical partial pressures of 1 mm Hg (1 torr) and 0.1 mm Hg (0.1 torr) respectively.
2. The lasing action takes place due to the transitions in neon atoms while the helium atoms help in the excitation of neon atoms.
3. The ends of the cavity are enclosed by two concave mirrors. One of the mirrors is 100% reflecting at the lasing frequency while the other is partially reflecting. Earlier, the mirrors used to be sealed

inside the glass but the tube does not last long since the seal gets eroded. Therefore now an external mirror arrangement is preferred.

4. The glass tube is closed by windows which are tilted at the Brewster angle. Such windows, allow the light waves with electric field in the plane of the paper to pass through without any reflection. The light waves with electric field perpendicular to the plane of the paper, are reflected away from the cavity.
5. With such windows, the output laser beam thus gets linearly polarized. Inside the gas cell, there are electrodes connected to the terminals of a dc power supply.

Figure 7



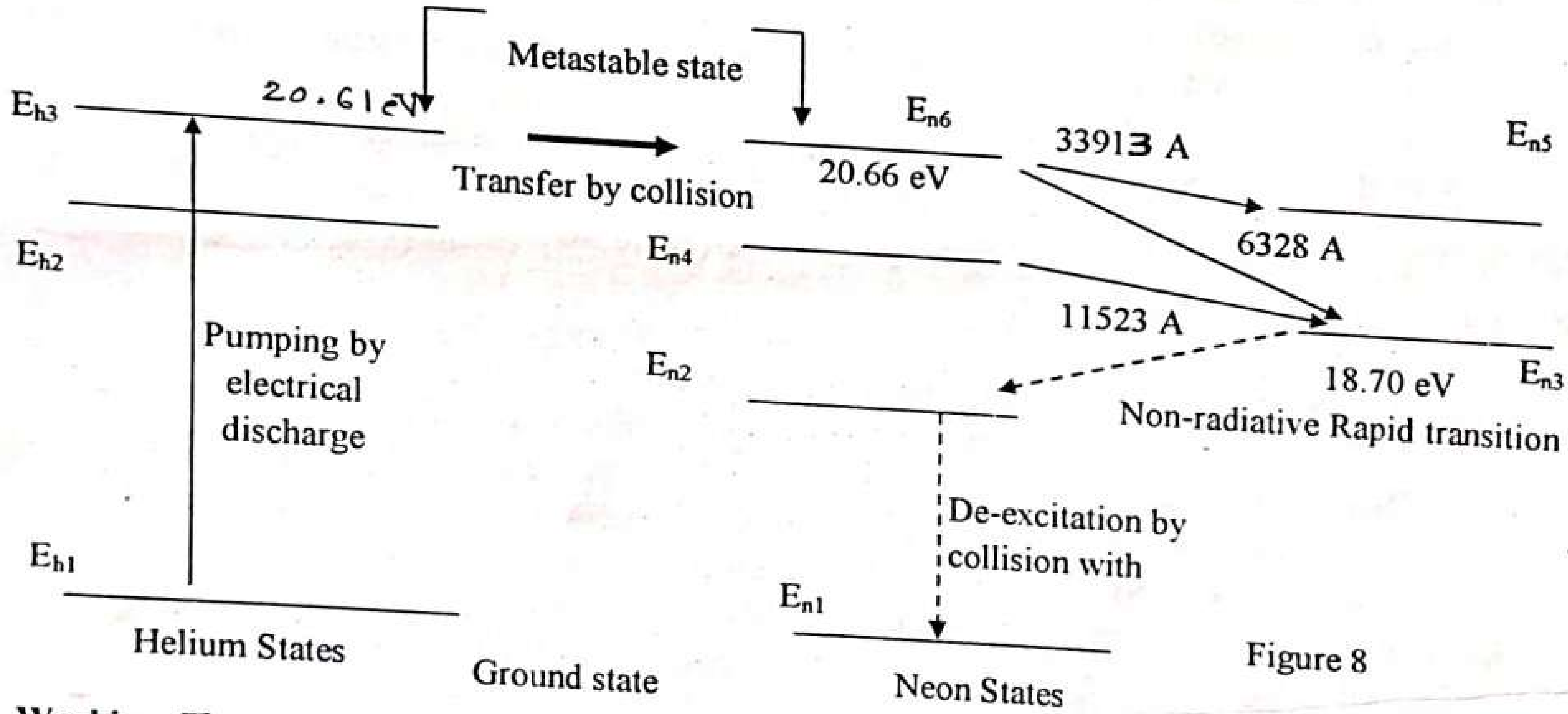


Figure 8

Working: The energy levels of *He* and *Ne* are shown in Fig. 8.

1. When an electrical discharge is passed through the gas, high energy electrons produced in the tube collide with the gas atoms.
2. As the concentration of helium atoms is higher, the probability of collision with He atoms is higher than that with the neon atoms.
3. These collisions excite the helium atoms to the higher energy states.
4. The helium atoms tend to accumulate in the metastable states E_{h2} and E_{h3} with respective lifetimes of 10^{-4} s and 5×10^{-6} s.
5. The energy levels E_{n4} and E_{n6} of neon atoms have almost the same energy as the levels E_{h2} and E_{h3} of the helium atoms.

6. Due to collisions between helium and neon atoms, the excited helium atoms excite the neon atoms to the levels, E_{n4} and E_{n6}.
7. We can represent this process as:

$$He^* (E_{h2}) + Ne (E_{n1}) \text{-------}\rightarrow He (E_{h1}) + Ne^* (E_{n4})$$
$$He^* (E_{h3}) + Ne (E_{n1}) \text{-------}\rightarrow He (E_{h1}) + Ne^* (E_{n6})$$

Where the letters in parentheses refer to the corresponding energy levels of gases.

8. The difference in the energy level of E_{h2} of He and E_{n4} of Ne is nearly 400 cm^{-1}. This difference in energy appears as the kinetic energy of the Ne atoms.
9. Direct excitations of Ne atoms to the levels E_{n4} and E_{n6} are also possible. However, due to less number of Ne atoms in the tube, it is less likely. Depending upon the energy levels involved in the transition, the major transitions are as follows:

a. **The 6328A transition:** When the lasing transition is from E_{n6} — E_{n3}, the wavelength of the produced laser beam is 6328A. The level E_{n6} is $3S_2$ and E_{n3} is $2P_4$. This is the most commonly obtained laser beam in He - Ne laser. The lifetime of E_{n3} is of the order of 10^{-8} sec while that of E_{n6} is 10^{-7} sec, hence, population inversion build up is possible between these two levels.

b. **The 33913A transition:** The transitions from E_{n6} — E_{n5} produce a laser beam of 33913A (or 3.3913 μm). The upper level is same in this case and in the 6328A transition.

c. **The 11523A transition:** This was the output wavelength of the first He - Ne laser. The transition from E_{n4} — E_{n3} produces photons beam of 11523A (or 1.1523 μm) wavelength.

Advantages:

1. The He-Ne laser system provides a continuous beam of laser light, unlike the ruby laser which provides a pulsed beam.
2. Further, it generates a low power laser beam. Typical small models generate a laser beam of power 1 mW and consume electric power of a few watts.
3. It is widely used in the laboratories where highly coherent and monochromatic sources are needed.
4. Used in supermarket scanners, printers, holography etc.

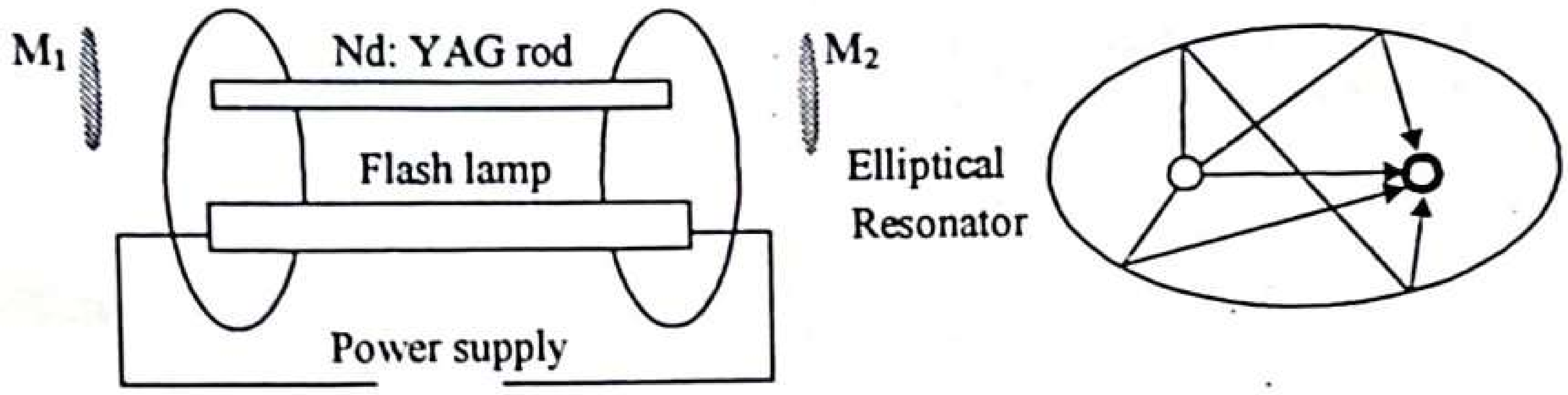
Nd-YAG laser:

Nd: YAG laser is one of the most popular types of solid state laser. It is a four level laser. Yttrium aluminium garnet, $Y_3Al_5O_{12}$, commonly called YAG is an optically isotropic crystal. Some of the Y^{3+} ions in the crystal are replaced by neodymium ions, Nd^{3+}. Doping concentration is typically of the order of 0.725 % by weight. The crystal atoms do not participate in the lasing action but serves as a host lattice in which the active center namely Nd^{3+} ions reside.

Construction: Fig. 9 illustrates a typical design of the laser.

1. It consists of an elliptically cylindrical reflector housing the laser rod along one of its focus lines and a flash lamp along the other focus line.
2. The light leaving one focus of the ellipse will pass through the other focus after reflection from the silvered surface of the reflector.
3. Thus the entire flash lamp radiation gets focused on the laser rod.
4. The YAG crystal rod is typically of 10 cm in length and 12 mm in diameter. The two ends of the laser rod are polished and silvered and constitute the optical resonator.

Figure 9



Working: The energy levels of the neodymium ion in YAG crystal are shown in Fig. 10.

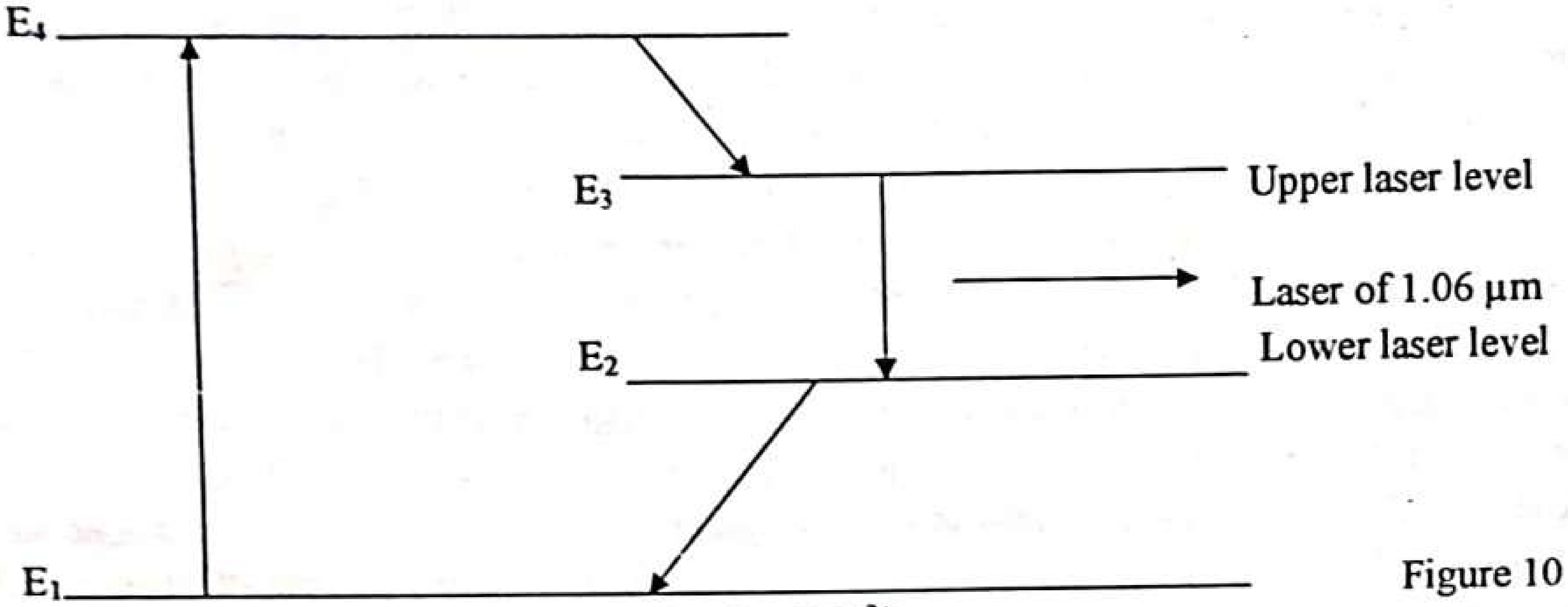


Figure 10

Energy levels of Nd^{3+}

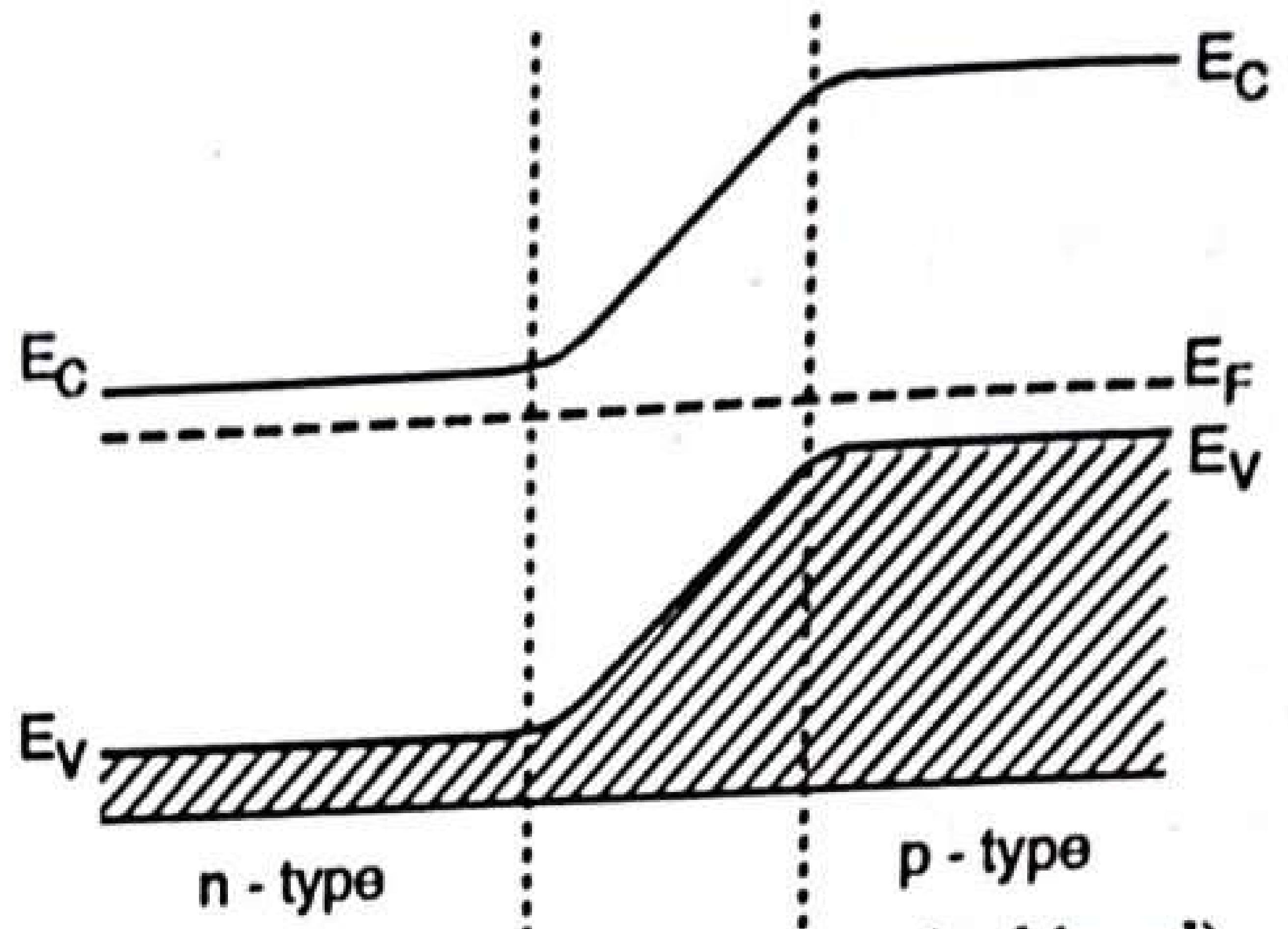
1. The energy level structure of the free neodymium atom is preserved to a certain extent because of its relatively low concentration. However, the energy levels are split and the structure is complex.
2. The pumping of the Nd^{3+} ions to upper states is done by a krypton arc lamp. The optical pumping with light of wavelength range 5000 to 8000 A excites the ground state Nd^{3+} ions to higher states.
3. The metastable state E_3 is the upper laser level, while the E_2 forms the lower laser level.
4. The upper laser level E_3 will be rapidly populated, as the excited Nd^{3+} ions quickly make downward transitions from the upper energy bands.
5. The lower laser level E_2 is far above the ground level and hence it cannot be populated by Nd^{3+} ions through thermal transitions from the ground level.
6. **Therefore, the population inversion is readily achieved between the E_3 level and E_2 level.**
7. The laser emission occurs in infrared (IR) region at a wavelength of about 10,600 A (1.06 μm).
8. As the laser is a four level laser the population inversion can be maintained in the face of continuous laser emission. Thus Nd: YAG laser can be operated in CW mode. An efficiency of better than 1% is achieved.

Advantages:

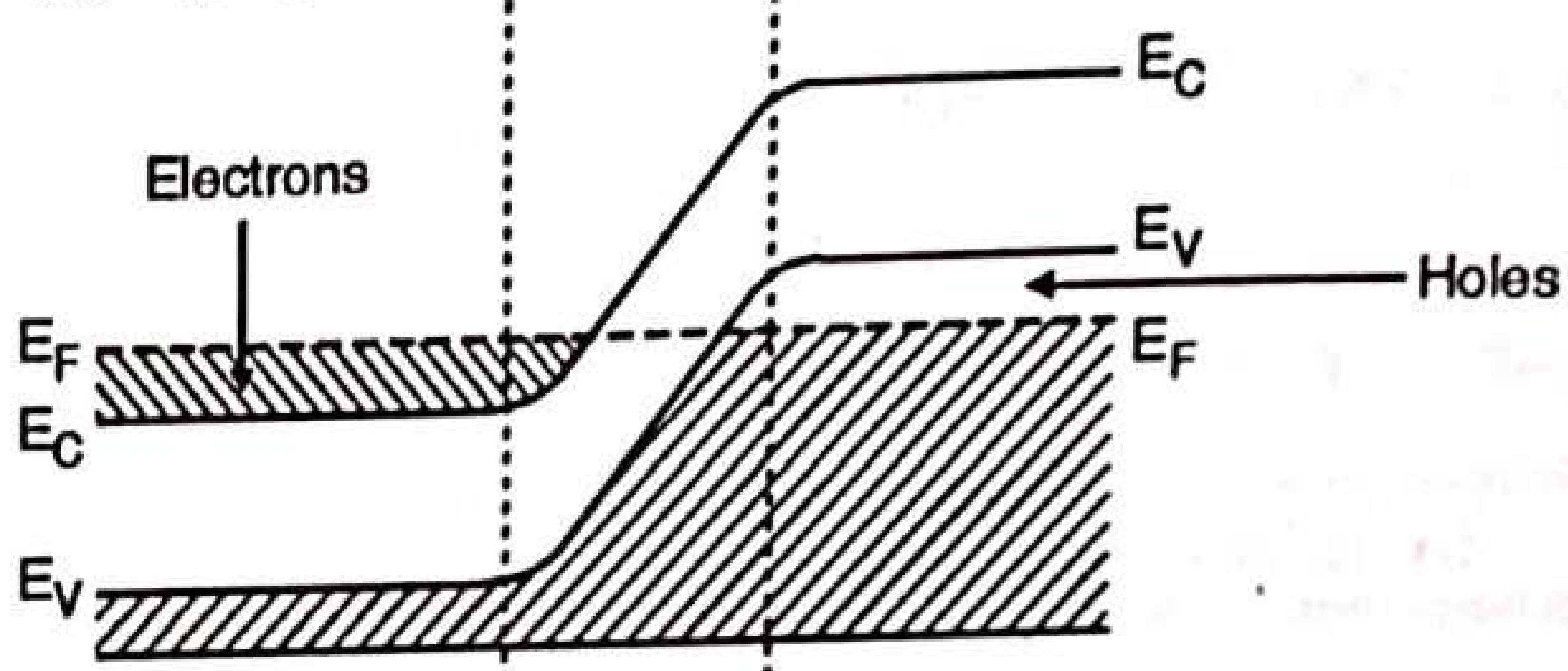
1. In optical communication systems, especially in atmospheric and free space communication links, the Nd: YAG laser is the most widely used laser.
2. This laser is also used for medical surgery, drilling, welding, etc.

4.13 Semiconductor Diode Laser

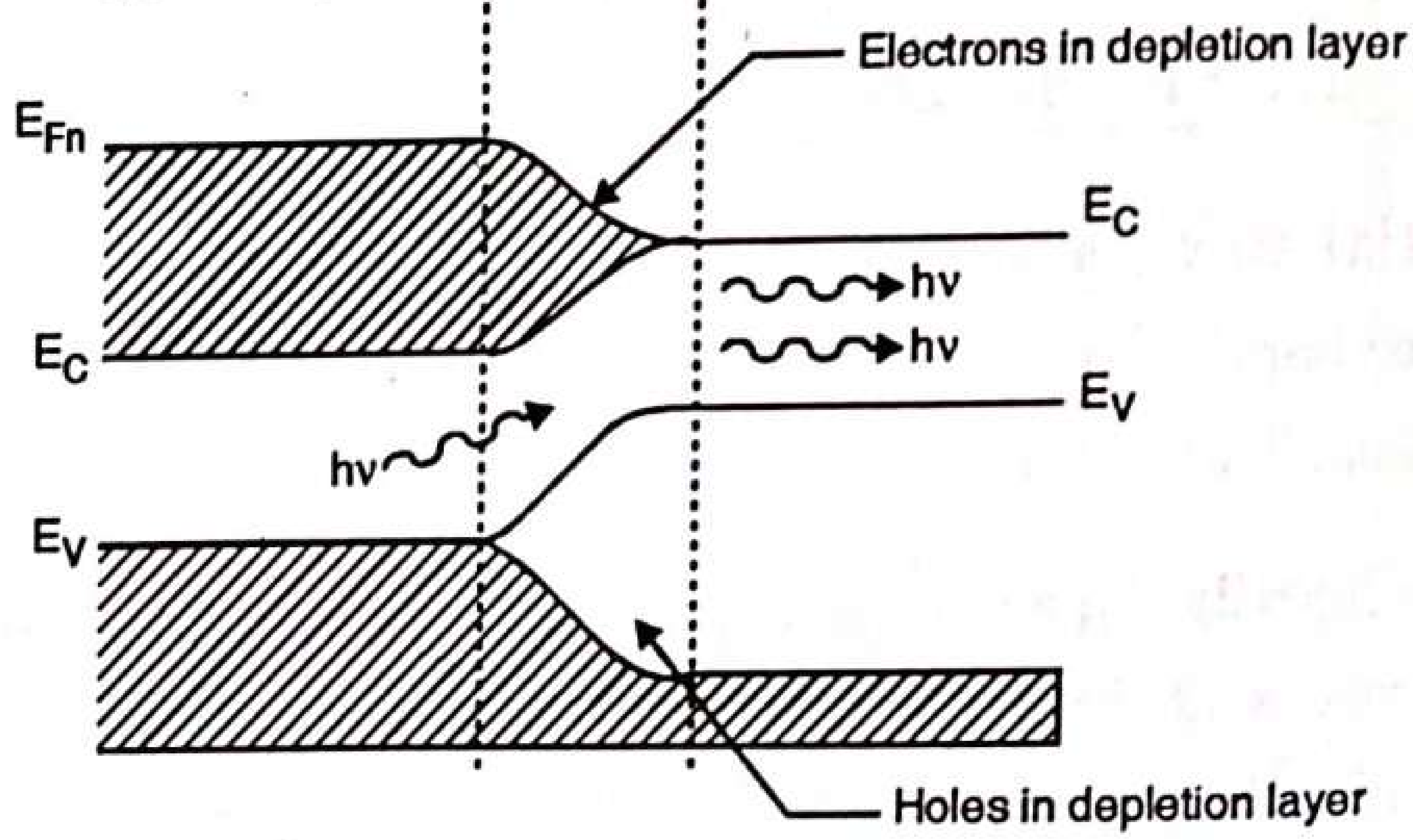
- Fig. 4.13.1(a) shows a scheme of an ordinary p-n junction semiconductor. The valence band in p-region has holes and conduction band in n region has free electrons. This is the condition when semiconductor is lightly doped.
- When it is heavily doped, we get some electrons shifted to conduction band and holes are seen in valence bands. But this does not create population inversion at all (Fig. 4.13.1(b)). The Fermi level on n-side is found on conduction band and on p-side it is found on valence band but it is in equilibrium at both sides.
- When a forward biased is applied, energy level diagram gets altered as shown in Fig. 4.13.1(c). Electrons from conduction band of n-type and holes from valence band of p-type are injected into depletion layer.



(a) Lightly doped p–n junction (unbiased)



(b) Heavily doped p–n junction (unbiased)



(c) A forward biased and heavily doped p-n junction above threshold value.

Fig. 4.13.1 : Energy bands in semiconductor laser

A threshold current (A minimum forward current) is defined below which electron-hole recombination will have spontaneous emission and p-n juction works as LED. A forward current above the threshold value, carrier concentration in depletion region will reach very high values which describes population inversion state. The emission of light due to recombination of electrons and holes will be stimulated emission and produces laser.

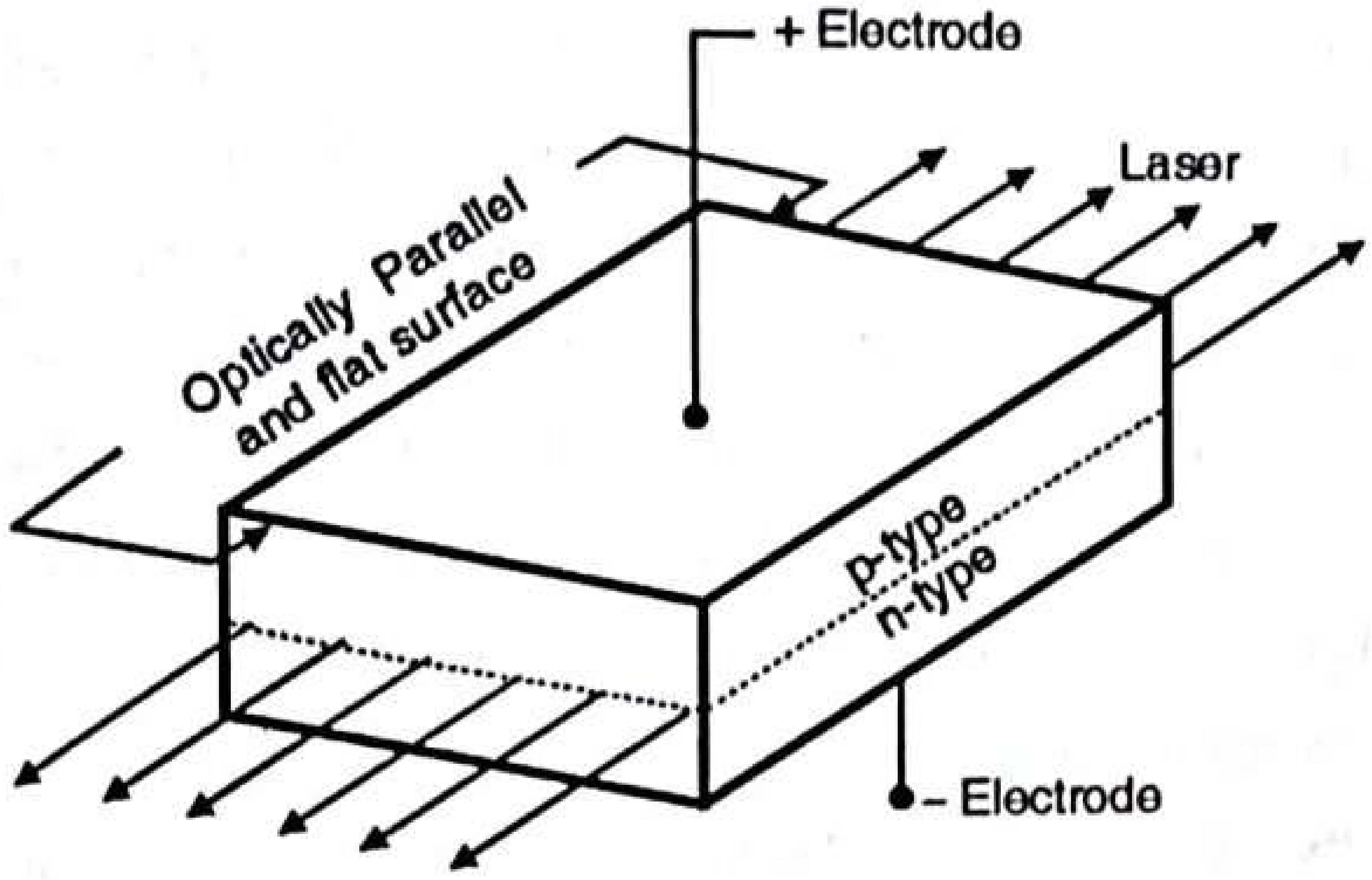


Fig. 4.13.2 : Schematic construction of a semiconductor diode

- Some of the semiconductor lasers and their wavelength.
  (a) GaAs : 8400 A° (at room temperature)
  (b) GaAsP : 6500 A° (at room temperature)
- **Merits**
  (a) Simple and compact.
  (b) Requires very little power and more efficient.
  (c) Output can be controlled by controlling the junction current.
  (d) Metastable state is not required.
- **Demerits**
  (a) Highly temperature sensitive.
  (b) Less monochromatic.
- **Applications**
  (a) Laser printers and copiers.
  (b) CD players.
  (c) Optical communication (as light source).

Holography:

In conventional photography, we obtain a two-dimensional record of the variation of intensity (square of the amplitude) of light received on a photographic plate from an object. It does not record the 'depth' of the object because the photographic film is not sensitive to phase variations in the waves. In 1948, Dennis Gabor developed a technique for recording both intensity and phase variations of light coming from various parts of the object, and thus producing three-dimensional images of the object. This technique is called holography, which is a Greek word meaning **the whole picture**. Gabor was awarded the 1971 Nobel Prize in physics for this discovery.

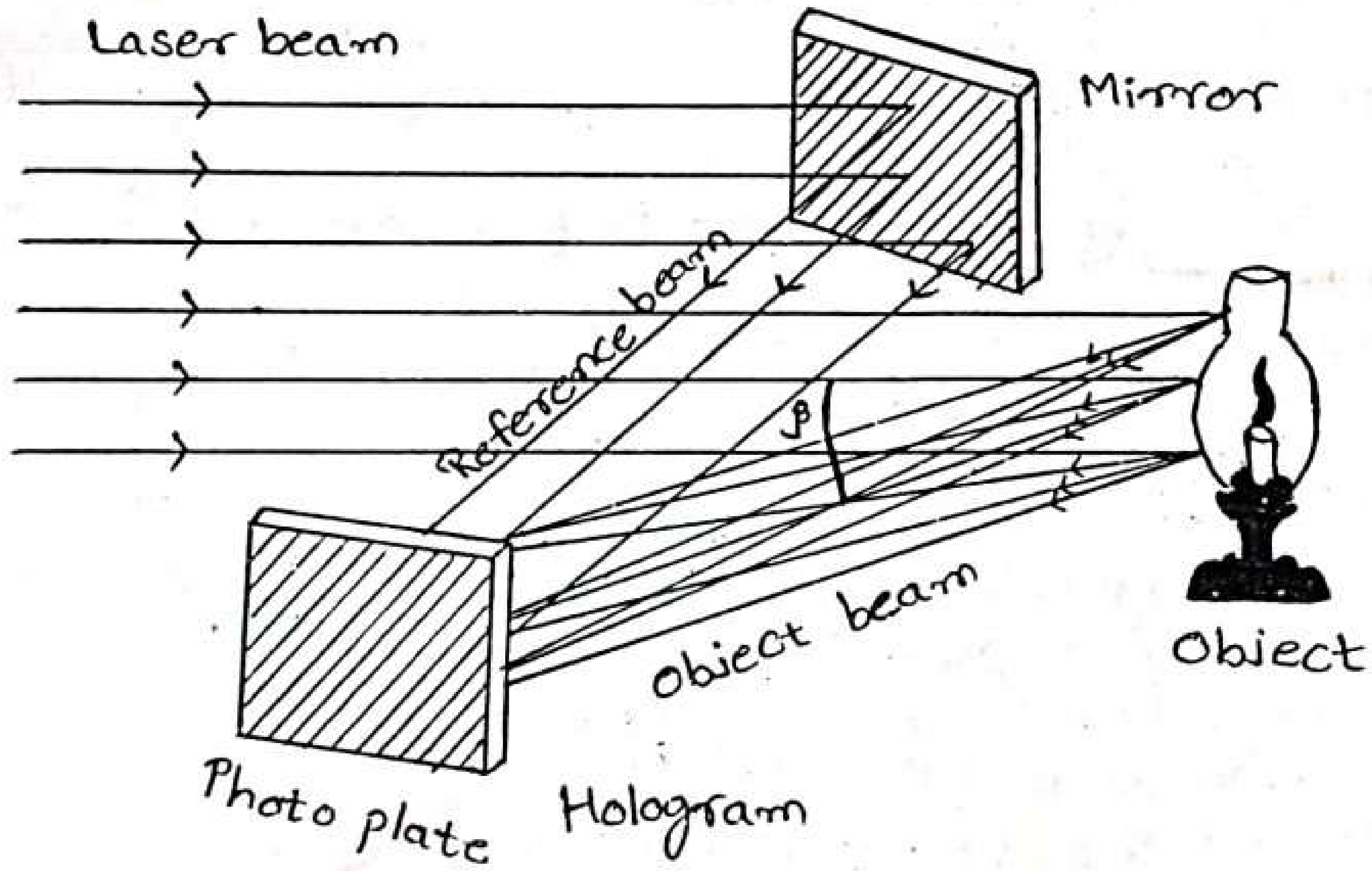


Figure : Generation of a hologram.

The principle of holography is illustrated in Figs.

1. Holography is a two-step process. In the first step, a laser beam from a source is split into two parts. One of the parts, called the reference wave, is allowed to fall directly on the photographic plate after reflection from a mirror M.
2. The second part, called the object wave, is allowed to fall on the object. A fraction of the light scattered from the object falls on the photographic plate.
3. The superposition of the reference wave and the scattered objective wave produces a complex interference pattern on the plate. The interference pattern records complete information about the intensity and phase of the wave from the object at each point.

5. The second step makes it possible to view the image of the object from the hologram. This is called the reconstruction process. The hologram is illuminated by a laser beam similar to the reference
6. The hologram acts as a diffraction grating and secondary waves from the hologram interfere constructively in certain directions and destructively in other directions. As a result, we get a real image of the object in front of the hologram and a virtual image behind the hologram. Both the images are in a complete three-dimensional form. The real image is inverted in depth.
7. It may be noted that a stable interference pattern can be obtained only if the object wave and the reference wave are coherent. The basic requirement for coherency is that the two waves originate from the same source. However, if the two waves are obtained from a conventional source, then they will not remain coherent when they reunite after travelling long distances because the *coherent length* for ordinary light is small. On the other hand, a laser beam has coherent length of the order of several hundred kilometers. Therefore, stable interference would be possible with laser light, even when the path difference between the two waves is large. Thus, holography could be realized in practice only after the discovery of laser.

Applications:

Holography has wide ranging applications in science and technology.

4.1 Introduction

What is relativity?

Consider a train moving with a speed of 60 km/hour. The train is observed by three observers.

(i) The first observer is standing at the station.

(ii) The second observer is moving in the direction of the train with a velocity of 20 km/hour.

(iii) The third observer is moving with a velocity of 20 km/hour in the opposite direction of the train.

The observations of the three observers are different as follows :

(i) The first observer would observe the velocity of the train as 60 km/hour.

(ii) The second observer would observe the velocity of the train as (60 – 20 =) 40 km/hour.

(iii) The third observer would observe the velocity of the train as (60 + 20 =) 80 km/hour.

The oldest theory of Physics is the Classical Physics or Newtonian Physics that deals with the absolute motion of an object considering space and time to be absolute and two separate entities. However, this concept failed to explain the motion with high velocities, very close to the velocity of light.

The development of theory of relativity by Einstein in 1905 revolutionalized the old concepts. It discards the concept of absolute motion and deals with objects and observers moving with high velocities ($\sim$ c) and relative velocities with respect to each other. This theory was developed in two steps and thus are divided into two parts.

(i) Einstein's Classical Theory of Relativity based on Classical Physics, *i.e.*, Newtonian mechanics.

(ii) Einstein's Special Theory of Relativity applicable to all laws of Physics.

4.2 Einsten's Classical Theory of Relativity (Newtonian Theory of Relativity)

Einstein initially developed his theory of relativity for classical physics, *i.e.*, Newtonian Mechanics. This is called Einstein's classical theory of relativity.

4.2.1 : Frame of Reference

The motion of an object can be described only with the help of a coordinate system. The coordinate system in such cases is known as the frame of reference. There are two types of frame of reference.

(1) Inertial frame of reference or unaccelerated frame

A frame of reference is said to be inertial when objects in this frame obey Newton's law of inertia and other laws of Newtonian mechanics. In this frame an object is not acted upon by an external force. It is at rest or moves with a constant velocity.

(2) Non inertial frame

A frame of reference which is in an accelerated motion with respected to an inertial frame of reference is called a non-inertial frame of reference. In such frame an object even without an external force acting on it, is accelerated. In non-inertial frame the Newton's laws are not valid.

Example : A ball placed the floor of a train moves to the rear if the train accelerates forward even though no forces act on it. In this case, the train moves in an inertial frame of reference and the ball is in a non-inertial frame of reference.

4.2.2 : Galilean Transformations

The transformation from one inertial frame of reference to another is called Galilean transformation. Knowing the laws of motion of an object in a reference system S, the laws of motion of the same object in another reference system S' can be derived.

Let us consider a physical event. An event is something that happens without depending on the reference frame used to describe it. Suppose a collision of two particles occur at a point (x, y, z) at an instant of t secs. We describe this event by the coordinates (x, y, z, t) in one frame of reference, say, in a laboratory on the earth. The same event observed from a different reference frame, *e.g.*, from an aircraft flying overhead would also be specified by a set of four coordinates in space and time (x', y', z', t') which is different from the earlier set of (x, y, z, t).

Consider now two observers O and P, where P travels with a constant velocity 'v' with respect to O along their common X-X' axis. Here E is the event specified by coordinates (x, y, z, t) and (x', y', z', t') in frames S and S' respectively.

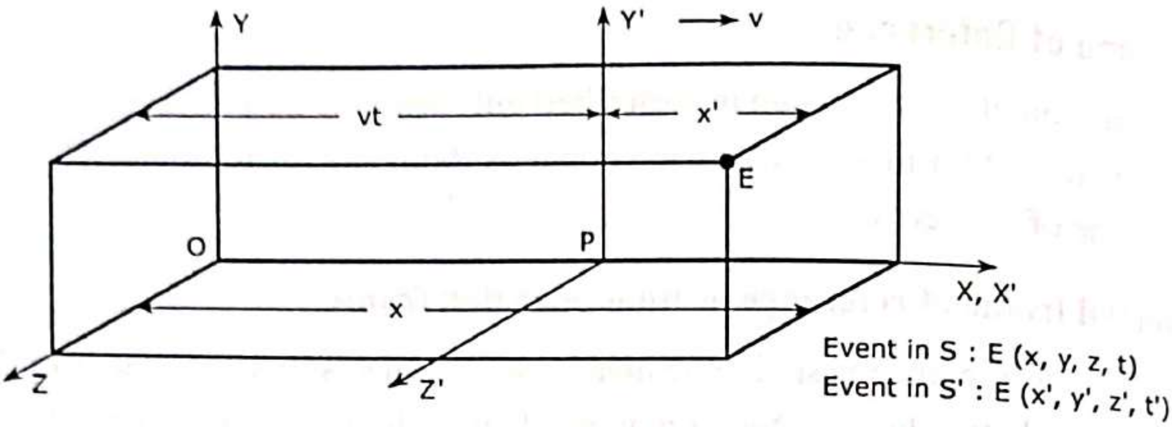


Fig. 4.1 : Frame of reference

(a) Galilean Coordinate Transformations

From Fig. 4.1, it is observed that

$$x' = x - vt, \; y' = y, \; z' = z \text{ and } t' = t \qquad \text{.................... (4.1}$$

These four equations are called Galilean coordinates transformations.

(b) Galilean Velocity Transformations

The velocity coordinates of the object in event E can be assigned as (u_x, u_y, u_z) an (u_x', u_y', u_z') in frame S and in frame S' respectively. Then from equation (4.1), it can b written as

$$u_x' = \frac{dx'}{dt'} = \frac{d}{dt}(x - vt)\frac{dt}{dt'} = \frac{dx}{dt} - v = u_x - v \qquad \text{as } \frac{dt}{dt'} = 1$$

Altogether, the Galilean velocity transformation are

$$u_x' = u_x - v, \quad u_y' = v_y, \quad u_z' = v_z \qquad \text{.................... (4.2}$$

(c) Galilean Acceleration Transformation

In inertial frames of reference S and S', the acceleration components remain th same. Thus,

$$a_x' = a_x, \quad a_y' = a_y, \quad a_z' = a_z \qquad \text{.................... (4.3}$$

4.3 Einstein's Special Theory of Relativity

Einstein observed that his Classical Theory of Relativity fails for very high spee (v ~ c) particles. This is due to the fact that in Newtonian mechanics, there is no limit, i principle, to the allowed speed of a particle. In 1905, he extended his Classical Theory o Relativity to include all the laws of Physics and Special Theory of Relativity was developed

The special theory of relativity deals with the problems in which one frame of reference moves with a constant linear velocity relative to another frame of reference.

4.3.1 : Postulates of Special Theory of Relativity

Einstein in his Special Theory of Relativity postulated that

(i) **All the fundamental laws of physics retain the same form in all the inertial frames of reference.**

(ii) **The velocity of light in free space is constant and is independent of the relative motion of the source and the observer in any frame of reference.**

4.3.2 : Einstein proved the following facts based on his theory of relativity

Let v be the velocity of a spaceship with respect to a given frame of reference where an observer makes his observations.

(a) All clocks on the spaceship will go slow by a factor $\sqrt{1-(v^2/c^2)}$.

(b) The mass of the spaceship increases by a factor $\left[1-(v^2/c^2)\right]^{-1/2}$.

(c) All objects on the spaceship will be contracted by a factor $\sqrt{1-(v^2/c^2)}$.

(d) The speed of a material object can never exceed the velocity of light.

(e) Mass and energy are interconvertible,

$$E = mc^2$$

(f) If two objects A and B are moving with velocities u and v respectively along the X-axis, the relative velocity of A with respect to B is given by

$$v_R = \frac{u-v}{1-(uv/c^2)}$$

Here, u and v are both comparable with the value of c.

4.3.3 : Lorentz Transformation of Space and Time

In Newtonian mechanics, the Galilean transformations expressed in equations (4.1), (4.2) and (4.3) relate the space and time coordinates in one inertial frame to those the other frame. However, these equations are not valid for cases where the object velocity v approaches the value of c, the velocity of light. The transformation equations apply for all velocities upto c and incorporate the invariance of the speed of light were developed in 1890 by Lorentz. These are known as Lorentz transformations.

Let us consider two inertial frames S and S' as shown in the Fig. 4.2. The frame S' moves with a velocity v with respect to S in the positive X direction.

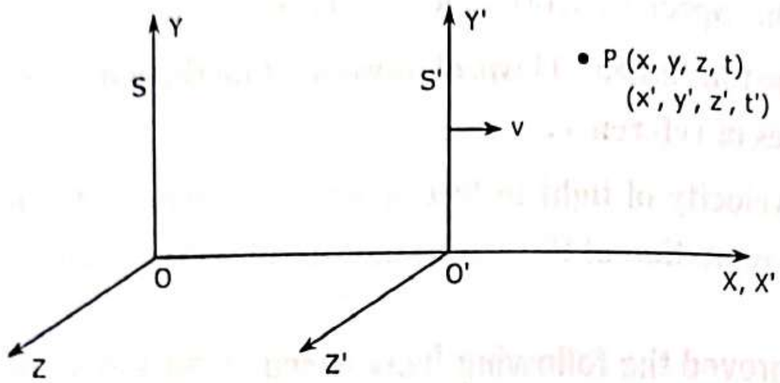


Fig. 4.2

Consider two observers O and O' situated at the origin in the frames S and S' respectively. Two coordinate systems coincide initially at the instant t = t' = 0. Suppose optical source is v kept at the common origin of the two frames. Let the source release a pulse at t = t' = 0 and at the same instant frame S' starts moving with a constant velocity v along +X direction, relative to frame S. This pulse reaches a point P with coordinates (x, y, z, t) and (x', y', z', t') in frames S and S' respectively.

Since S' is moving along +X direction with respect to S, the transformation equation of x and x' can be written as

$$x' = k(x - vt) \qquad \text{.................... (4.4)}$$

where, k is the constant of proportionality.

The inverse relation can be written as,

$$x = k(x' + vt') \qquad \text{.................... (4.5)}$$

Putting equation (4.4) in equation (4.5), we can write

$$x = k[k(x - vt) + vt']$$

$$\therefore \quad t' = kt - \frac{kx}{v}\left(1 - \frac{1}{k^2}\right) \qquad \text{.................... (4.6)}$$

Now, according to the second postulate of relativity, the speed of light c remains constant. So the velocity of the light pulse spreading out from the common origin observed by observers O and O' should be the same

$$x = ct \qquad \text{.................... (4.7)}$$

$$x' = ct'$$

Substituting equation (4.7) in equation (4.4) and equation (4.5), we have

$$ct' = k(c-v)t \qquad \text{.................... (4.8)}$$

and

$$ct = k(c+v)t' \qquad \text{.................... (4.9)}$$

Multiplying equations (4.8) with equation (4.9), we have .

$$\therefore \qquad k^2 = \frac{c^2}{c^2 - v^2}$$

$$\therefore \qquad k = \pm\frac{1}{\sqrt{1-(v^2/c^2)}} \qquad \text{.................... (4.10)}$$

and

$$1 - \frac{1}{k^2} = \frac{v^2}{c^2}$$

Using equations (4.10) in equation (4.4), we have

$$x' = \frac{x - vt}{\sqrt{1-(v^2/c^2)}} \qquad \text{.................... (4.11)}$$

Substituting equations (4.10) and (4.11), we have

$$t' = \frac{t-(xv/c^2)}{\sqrt{1-(v^2/c^2)}} \qquad \text{.................... (4.12)}$$

Hence, if the frame S' moves with a velocity v in +X direction with respect to the frame S, the transformation equations are,

$$x' = \frac{x - vt}{\sqrt{1-(v^2/c^2)}}, \quad y' = y, \quad z' = z, \quad t' = \frac{t-(xv/c^2)}{\sqrt{1-(v^2/c^2)}} \qquad \text{.............. (4.13)}$$

On the otherhand, if the frame S moves with a velocity v in – X direction with respect to the frame S', we get the inverse transformation equations as

$$x = \frac{x' + vt'}{\sqrt{1-(v^2/c^2)}}, \quad y = y', \quad z = z', \quad t = \frac{t'-(x'v/c^2)}{\sqrt{1-(v^2/c^2)}} \qquad \text{............... (4.14)}$$

If the speed of the moving frame is much smaller than the velocity of light, *i.e.*, if $v \ll c$, the Lorentz transformation equations reduce to Galilean transformation equations.

4.4 Time Dilation

The meaning of time dilation is extension of time. Time dilation is a difference in the elapsed time measured by two clocks due to a relative motion between them. To explain it let us consider two frames of reference S and S' with S' moving with a velocity v along X direction with respect to S as shown in Fig. 4.3. Imagine a gun placed at a fixed position P (x', y', z') in the frame S'. Suppose it fires two shots at instants t_1' and t_2' measured by the observer O' in the frame S'.

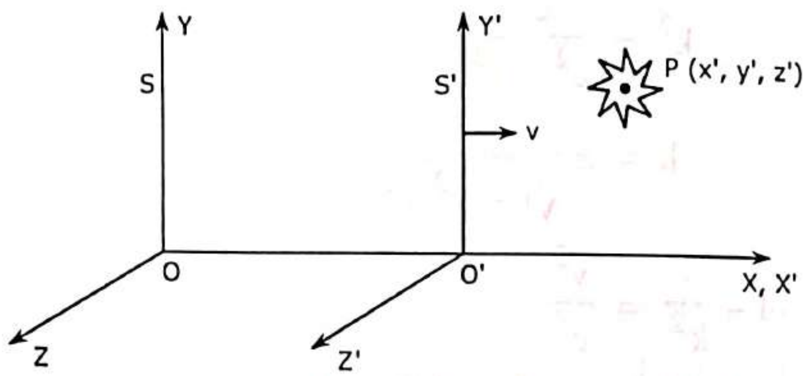


Fig. 4.3 : Time dilation

The time interval $(t_2' - t_1')$ of the two shots measured by O' at rest in the moving frame S' is called the proper time interval and is given by

$$T_O = t_2' - t_1' \qquad \text{.................... (4.15)}$$

As the motion between the two frames is relative, we may assume that the frame S is moving with velocity – v along the – X direction relative to frame S'. In frame S, the observer O who is at rest hears these two shots at different times t_1 and t_2.

The time interval appears to him is given by

$$t = t_2 - t_1 \qquad \text{.................... (4.16)}$$

From inverse Lorentz transformation equations, we get

$$t_1 = \frac{t_1' + (vx'/c^2)}{\sqrt{1-(v^2/c^2)}} \qquad \text{.................... (4.17)}$$

$$t_2 = \frac{t_2' + (vx'/c^2)}{\sqrt{1-(v^2/c^2)}} \qquad \text{.................... (4.18)}$$

Substituting equations (4.17) and (4.18) in equation (4.16), we get

$$T = \frac{t_2' - t_1'}{\sqrt{1-(v^2/c^2)}} \qquad \text{.................... (4.19)}$$

Using equation (4.15) in equation (4.19), we have

$$T = \frac{T_0}{\sqrt{1-(v^2/c^2)}} \qquad \text{.................... (4.20)}$$

which shows that $T > T_0$.

Here, T_o is called the **proper time which is defined as the time measured in the frame of reference in which the object is at rest.**

This verifies that the actual time interval in the moving frame appears to be lengthened by a factor $\frac{1}{\sqrt{v^2 - c^2}}$ when it is measured by an observer in the fixed frame, v being the relative velocity between the two frames.

4.5 Length Contraction

In classical mechanics the length of an object is independent of the velocity of the observer moving relative to the object. However, in the theory of relativity, the length of an object depends on the relative velocity between the observer and the object.

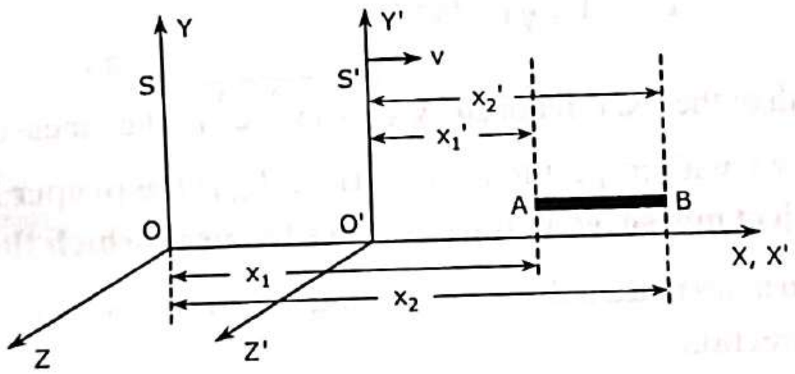


Fig. 4.4

To explain this, let us consider two inertial frames S and S' with S' moving with a velocity v in the X direction with respect to S.

Let a rod AB be at rest the moving frame S'. Its actual length is L_0 at any instant as measured by the observer O' also at rest in the frame S'. So,

$$L_0 = x_2' - x_1' \qquad \text{.................... (4.21)}$$

where, x_1' and x_2' are the x coordinates of the rod in frame S' as shown in the Fig. 4.4.

At the same time, the length of AB measured by an observer O in the stationary frame S is given by

$$L = x_2 - x_1 \quad \text{.................... (4.22)}$$

x_1 and x_2 being the x coordinates of the rod in frame S.

From Lorentz transformation,

$$x_1' = \frac{x_1 - vt}{\sqrt{1-(v^2/c^2)}} \quad \text{.................... (4.23)}$$

$$x_2' = \frac{x_2 - vt}{\sqrt{1-(v^2/c^2)}} \quad \text{.................... (4.24)}$$

Substituting equations (4.23) and (4.24) in equation (4.21), we get the actual length as

$$L_o = \frac{x_2 - x_1}{\sqrt{1-(v^2/c^2)}} \quad \text{.................... (4.25)}$$

Using equation (4.22) in equation (4.25), we have

$$L_o = \frac{L}{\sqrt{1-(v^2/c^2)}}$$

$$\therefore \quad L = L_o \sqrt{1-(v^2/c^2)} \quad \text{.................... (4.26)}$$

Thus, the length of the rod is reduced by $\sqrt{1-(v^2/c^2)}$ when measured by an observer moving with velocity v with respect to the rod. Here, L_o is the **proper length defined as the length of the object measured in the reference frame in which the object is at rest.**

The contraction takes place only along the direction of motion and remains unchanged in a perpendicular direction.

4.6 Einstein's Mass-Energy Relation

In classical mechanics, the mass of a particle is independent of its velocity but in Einstein's special theory of relativity, the mass of a moving object depends upon its velocity and is given by

$$m = \frac{m_o}{\sqrt{1-(v^2/c^2)}}$$

where, m_o is the rest mass and v is the velocity of the moving body and c is the velocity of light.

The increase in the energy of a particle by the applications of force may be estimated by the work done on it.

If a particle is displaced by a distance dx on the application of a force F, the kinetic energy dE generated and stored in it is given by the work done,

$$dE = dW = F\,dx \qquad \text{.................... (4.27)}$$

Now, the force is defined as the time rate of change of momentum of the particle, by Newton's second law. Hence,

$$F = \frac{d(mv)}{dt} \qquad \text{.................... (4.28)}$$

Where, m is the mass of the particle and v is its velocity with which it moves on the application of the force F.

Thus, combining equations (4.27) and (4.28), we get

$$dE = \frac{d(mv)}{dt} \cdot dx$$

$$dE = \frac{dx}{dt}\, d(mv) = v\,[m\,dv + v\,dm]$$

$$\therefore \quad dE = mv\,dv + v^2\,dm \qquad \text{.................... (4.29)}$$

Again, $$m = \frac{m_o}{\sqrt{1-(v^2/c^2)}}$$

So, $$m^2 = \frac{m_o^{\,2}}{1-(v^2/c^2)}$$

$$m^2 c^2 - m^2 v^2 = m_o^{\,2} c^2 \qquad \text{.................... (4.30)}$$

Differentiating equation (4.30), with m_o and c constants, we have

$$2m\,dm\,c^2 - 2m\,dm\,v^2 - 2v\,dv\,m^2 = 0$$

$$\therefore \quad dm\,c^2 = v^2\,dm + mv\,dv \qquad \text{.................... (4.31)}$$

Substituting equation (4.31) in equation (4.29), we get

$$dE = dm\,c^2 \qquad \text{.................... (4.32)}$$

Showing that the **change in kinetic energy is directly proportional to the change in mass of the particle.**

From equation (4.30), it is obvious that for a rest object v = 0 and mass $m = m_0$, the rest mass.

If the particle moves with a velocity v, its mass become m and its kinetic energy becomes E_k. Therefore, integrating equation (4.32), we get

$$\int_0^{E_k} dE = c^2 \int_{m_o}^{m} dm$$

$$\therefore \quad E_k = c^2 (m - m_o)$$

$$\therefore \quad E_k = mc^2 - m_o c^2 \qquad \text{.................... (4.33)}$$

$$\text{or} \quad mc^2 = E_k + m_o c^2 \qquad \text{.................... (4.34)}$$

Here, mc^2 is the total energy, $m_o c^2$ is the rest mass energy and E_k is its kinetic energy. Hence, we write

$$E = E_k + m_o c^2 \qquad \text{.................... (4.35)}$$

$$\text{and} \quad \mathbf{E = mc^2} \qquad \text{.................... (4.36)}$$

Equation (4.36) is known as ***Einstein's mass-energy relation***.

4.7 Important Points to Remembers

1. Space and Time transformation relations

	Galilean transformation	*Lorentz transformation*	*Inverse Lorentz transformation*
X - coordinates	$x' = x - vt$	$x' = \frac{x - vt}{\sqrt{1-(v^2/c^2)}}$	$x = \frac{x' - vt}{\sqrt{1-(v^2/c^2)}}$
Y - coordinates	$y' = y$	$y' = y$	$y = y'$
Z - coordinates	$z' = z$	$z' = z$	$z = z'$
Time coordinate	$t' = t$	$t' = \frac{t-(vx/c^2)}{\sqrt{1-(v^2/c^2)}}$	$t = \frac{t' + (vx'/c^2)}{\sqrt{1-(v^2/c^2)}}$

2. Time dilation : $T = \frac{T_0}{\sqrt{1-(v^2/c^2)}}$

3. Length contraction : $L = L_0 \sqrt{1-(v^2/c^2)}$

4. Einstein's mass energy relation : $E = mc^2$ and $E_k = mc^2 - m_o c^2$.

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Electrodynomics

Del operator :-

(1) As per conversions any position vector is defined as:

$$r = \hat{i}x + \hat{j}y \text{ or } \hat{k}z$$

or

$$a_x.x + a_y.y + a_z.z.$$

where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors in the specific directions, x, y, z axis

(2) The differential operator ∇ (del) is defined as

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

OR

$$\nabla = a_x\frac{\partial}{\partial x} + a_y\frac{\partial}{\partial y} + a_z\frac{\partial}{\partial z}.$$

(3) Del is not a vector in itself, but when operates on a scalar function it provides the resultant as vector. The del operator can operate in different ways

(3) For example, when it acts on a scalar function 'F,' the resultant ∇F is called the 'gradient of a scalar function. when it acts on a vector function $\vec{A}$ via the dot product resultant is $\vec{\nabla} \cdot \vec{A}$ which is called 'divergence of vector $\vec{A}$. whereas when it acts via cross product $\vec{\nabla} \times \vec{A}$, the resultant is a vector and called 'curl of a vector $\vec{A}$.

Significance of Gradient

(1) A gradient is a directional derivative. In a simple terms it is the rate of change of a function' in a specified directions.

(2) The gradient of a scalar function is a vector quantity

(3) Mathematically, if $\phi(x,y,z)$ is differentiable at each point (x,y,z) in a certain region of space then.

$$\nabla \phi(x,y,z) = \hat{i}\frac{\partial \phi}{\partial x} + \hat{j}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z}$$

OR

$$\nabla \phi(x,y,z) = a_x \frac{\partial \phi}{\partial x} + a_y \frac{\partial \phi}{\partial y} + a_z \frac{\partial \phi}{\partial z}$$

(4) If we think of derivative of a function of one variable we notice that it simply tells us how fast the function varies if we move a small distance. It means gradient is the rate of change of a quantity with distance.

Significance of Divergence.

(1) It is known that divergence of vector field $\vec{A}$ is expressed as $\vec{\nabla}.\vec{A}$. It is given by

$$\vec{\nabla}.\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}.$$

(2) clearly the divergence of a vector is Scalar. Also, the diverguence of a scalar cannot be obtained.

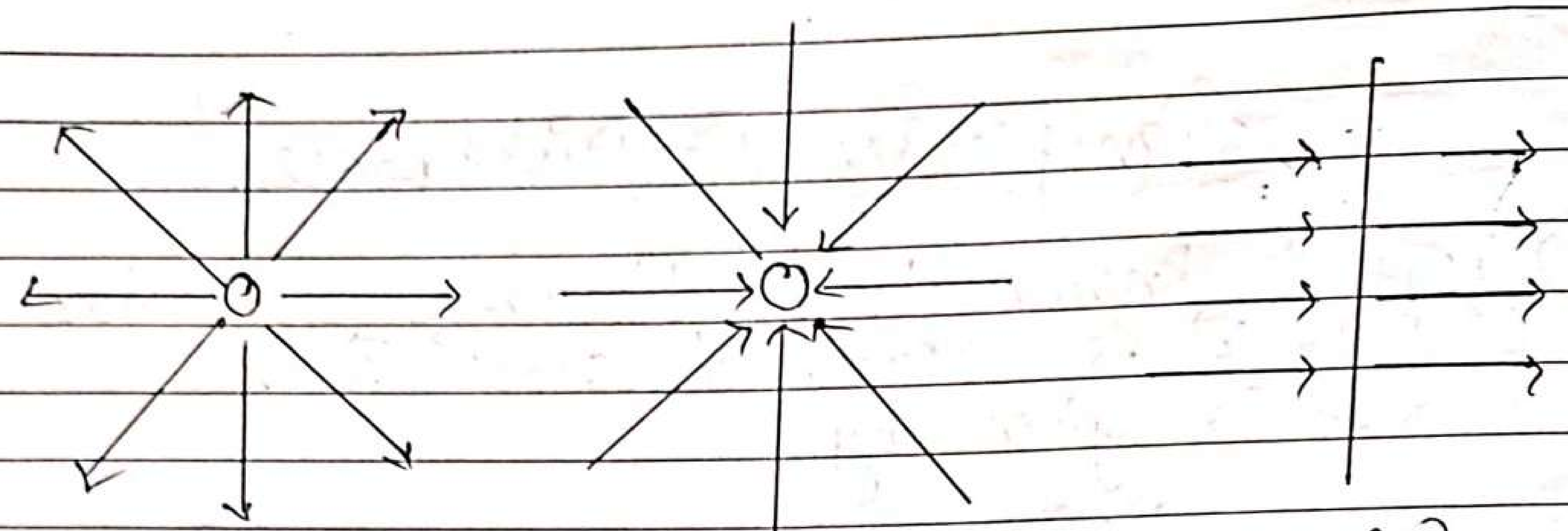
(3) The Simple Meaning can be expressed by considering net flux $\oint \vec{A}.d\vec{s}$ of a vector field $\vec{A}$ from a closed surface S.

(4) The divergence of $\vec{A}$ is defined as net outward flux unit volume over a closed surface.

(5) In fig (1) at the point O the divergence of vector field is positive as the vector spreads out which also represents source.

(6) In fig (2) the field is converging and hence divergence at point O is negative which represents sink.
In fig (3) one can notice that divergence of vector field is zero.

(7) A water fountain, or a tyre which has just been punctured by a nail where air is expanding and forming a net outflow are some more examples which may be used to represent divergence.



(1) Positive Divergence (Source) (2) Negative Divergence (Sink) (3) Zero Divergence (Incompressible)

Significance of curl of a vector.

(1) Curl is simply defined as circulation per unit area where a closed path is vanishingly small. It is represented as $\vec{\nabla} \times \vec{A}$ for a vector field $\vec{A}$.

(2) Mathematically,

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

OR

$$\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

(3) curl is a vector quantity as it has magnitude and direction both.
In order to make its meaning clear, we consider the circulation of a vector field $\vec{A}$ around a closed path $\oint \vec{A} \cdot d\vec{l}$.

(4) It is evident that the curl of $\vec{A}$ is a rotational vector. Its Magnitude would be the Maximum circulation of $\vec{A}$ per unit area.

(5) If $\vec{\nabla} \times \vec{A} = 0$ the vector field is irrotational.

Methods of Solving Sums.

Q.1) If $\phi(x,y,z) = 3x^2y - y^3z^2$, find gradient of the scalar function i.e. $\nabla\phi$ at point $(1,-2,-1)$.

Ans = The gradient of ϕ is given by

$$\nabla\phi = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(3x^2y - y^3z^2)$$

$$\nabla\phi = \hat{i}\frac{\partial(3x^2y - y^3z^2)}{\partial x} + \hat{j}\frac{\partial(3x^2y - y^3z^2)}{\partial y} + \hat{k}\frac{\partial(3x^2y - y^3z^2)}{\partial z}$$

$$\nabla\phi = 6xy\,\hat{i} + (3x^2 - 3y^2z^2)\hat{j} \cancel{+} - 2y^3z\,\hat{k}$$

at point $(1,-2,-1)$

$$\nabla\phi = 6(1)(-2)\,\hat{i} + (3(1)^2 - 3(-2)^2(-1)^2)\hat{j} + 2(-2)^3(-1)\,\hat{k}$$
$$= -12\hat{i} - 9\hat{j} - 16\hat{k}$$

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Q.2) If $\vec{A} = x^2 z\hat{i} - 2y^2z^2\hat{j} + xy^2z\hat{k}$ find divergence of a vector at point $(1,-1,1)$

Ans = The divergence of $\vec{A}$ is given by

$$\vec{\nabla}\cdot\vec{A} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left(x^2z\hat{i} - 2y^2z^2\hat{j} + xy^2z\hat{k}\right)$$

$$\vec{\nabla}\cdot\vec{A} = \frac{\partial (x^2z)}{\partial x} - \frac{\partial (2y^2z^2)}{\partial y} + \frac{\partial}{\partial z} xy^2z$$

$$\vec{\nabla}\cdot\vec{A} = 2xz - 4y^2z^2 + xy^2$$

at point $(1,-1,1)$

$$\vec{\nabla}\cdot\vec{A} = 2(1)(1) - 4(-1)^2(1)^2 + 1(-1)^2$$
$$= 7.$$

Q.3) Determine the curl of the following vector.

$$\vec{A} = (2x^2+y^2)\hat{i} + (xy-y^2)\hat{j}$$

Ans = The curl of $\vec{A}$ is given by.

$$\vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x^2+y^2 & xy-y^2 & 0 \end{vmatrix}$$

$$\vec{\nabla}\times\vec{A} = \hat{i}\left[\frac{\partial}{\partial y}0 - \frac{\partial}{\partial z}(xy-y^2)\right] - \hat{j}\left[\frac{\partial}{\partial x}0 - \frac{\partial (2x^2+}{\partial z}\right.$$
$$+\hat{k}\left[\frac{\partial}{\partial x}(xy-y^2) - \frac{\partial}{\partial y}(2x^2+y^2)\right]$$

$\therefore \vec{\nabla} \times \vec{A} = -y\hat{k}$

Q.4) <u>Imp</u>

Show that divergence of the curl of vector is zero.

Ans let $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

$$\vec{\nabla} \times \vec{F} = \hat{i}\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) - \hat{j}\left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z}\right) + \hat{k}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$$

Now, ~~$\nabla\cdot(\nabla \times F)$~~ div (curl F) $= \vec{\nabla}\cdot(\vec{\nabla} \times \vec{F})$

$$\therefore \vec{\nabla}\cdot(\vec{\nabla} \times \vec{F}) = \frac{\partial}{\partial x}\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) - \frac{\partial}{\partial y}\left(\frac{\partial F_z}{\partial x} - \frac{\partial f_y}{\partial z}\right) + \frac{\partial}{\partial z}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$$

$$= \cancel{\frac{\partial^2 F_z}{\partial x \partial y}} - \cancel{\frac{\partial^2 F_y}{\partial x \partial z}} - \cancel{\frac{\partial^2 F_z}{\partial y \partial x}} + \cancel{\frac{\partial^2 F_x}{\partial y \partial z}} + \cancel{\frac{\partial^2 F_y}{\partial z \partial x}} - \cancel{\frac{\partial^2 F_x}{\partial z \partial y}}$$

$$= 0$$

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Maxwell's First Equation in Differential Form.

(1) let us assume an arbitrary surface 'S' bounding an arbitrary volume 'V' in a dielectric Medium.
For any dielectric medium the total charge density is the sum of the free charge density (ρ_f) and polarised charge density (ρ_p).

(2). The total flux ϕ crossing the closed surface is equal to the total charge enclosed by that surface (Gauss law of Total Normal Electric Induction).

$$\therefore \oint_S \vec{E} \cdot \vec{ds} = \frac{Q}{\varepsilon_0} \quad \text{—— (1)}$$

$$\therefore \oint_S \vec{E} \cdot \vec{ds} = \frac{q_F + q_P}{\varepsilon_0}$$

$$\therefore \oint_S \vec{E} \cdot \vec{ds} = \frac{1}{\varepsilon_0} \oint_V (\rho_f + \rho_p)\, dv \quad \text{—— (2)}$$

Now, in electrodynamics $\rho_p = -\nabla \vec{P}$ where $\vec{P}$ is polarisation vector.

$$\therefore \oint_S \vec{E} \cdot \vec{ds} = \frac{1}{\varepsilon_0} \oint_V (\rho_f - \nabla \vec{P})\, dv \quad \text{—— (3)}$$

(3) using Gauss Divergence theorem to convert surface integration to volume integration,

i.e. $\oint_S \varepsilon_0 \vec{E}\, ds = \oint_V \nabla (\varepsilon_0 E)\, dv$.

$\therefore \oint_V \vec{\nabla} (\varepsilon_0 \vec{E})\, dv = \oint_V (\rho_f - \nabla \vec{P})\, dv$

$\therefore \vec{\nabla} \varepsilon_0 \vec{E} = \rho_f - \nabla \vec{P}$

$\vec{\nabla} \varepsilon_0 \vec{E} + \nabla \vec{P} = \rho_f$

$\vec{\nabla} (\varepsilon_0 \vec{E} + \vec{P}) = \rho_f$

As electric Displacement vector,

$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$.

$\therefore \vec{\nabla} \vec{D} = \rho_f$.

This is Maxwell's first Equation in Differential form.

Maxwell's first Equation in Integral form.

(1) The differential form of Maxwell's First equation is given by

$\vec{\nabla} \cdot \vec{D} = \rho$

Taking integration over complete volume V.

$\therefore \oint_V \vec{\nabla} \cdot \vec{D}\, dv = \oint_V \rho\, dv$.

(2) using Gauss Divergence theorem

$$\oint_V \vec{\nabla} \cdot \vec{D}\, dv = \oint_S \vec{D} \cdot ds$$

(1)

$$\therefore \oint_S \vec{D} \cdot ds = \int_V \rho_f\, dv$$

$$\therefore \oint_S \vec{D} \cdot ds = q_f$$

This is Maxwell's First Equation in integral form.

(2)

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Maxwell's Second Equation

(1) As the Magnetic lines of force are closed (or go to infinity) the number of Magnetic lines of flux entering any Surface is exactly the same as leaving the surface

(2) The total flux of the Magnetic field over a closed surface is given by

$$\oint_S \vec{B} \cdot \vec{ds} = 0 \quad \text{—— (1)}$$

(3) Using Gauss Divergence theorem convert surface integral to volume integral

$$\therefore \oint_S \vec{B} \cdot \vec{ds} = \oint_V \nabla \cdot B \, dv \quad \text{—— (2)}$$

$$\therefore \vec{\nabla} \cdot \vec{B} = 0 \quad \text{—— (3)}$$

This is known as the differential form of Maxwell's Second Equation.

(4) Equation (3) shows that as opposed to free electric charge, free Magnetic poles cannot exist anywhere, as the surface integral of B vanishes over any arbitrary surface., i.e. isolated Magnetic monopoles do not exist.

(5) Magnetic poles always exist in pairs and free magnetic north poles or South poles do not exist.

Maxwell's Second Equation in Integral form.

(1) Differential form of Maxwell's second eqn is given by.

$$\vec{\nabla} \cdot \vec{B} = 0$$

(2) Integrating with respect to volume and Again using Gauss Divergence theorem

$$\int_V \vec{\nabla} \cdot \vec{B} \, dv = 0$$

$$\therefore \oint_S \vec{B} \cdot \vec{ds} = 0$$

This eqn is known as integral form of Maxwell's Second equation.

Maxwell's Third Equation in Differential Form

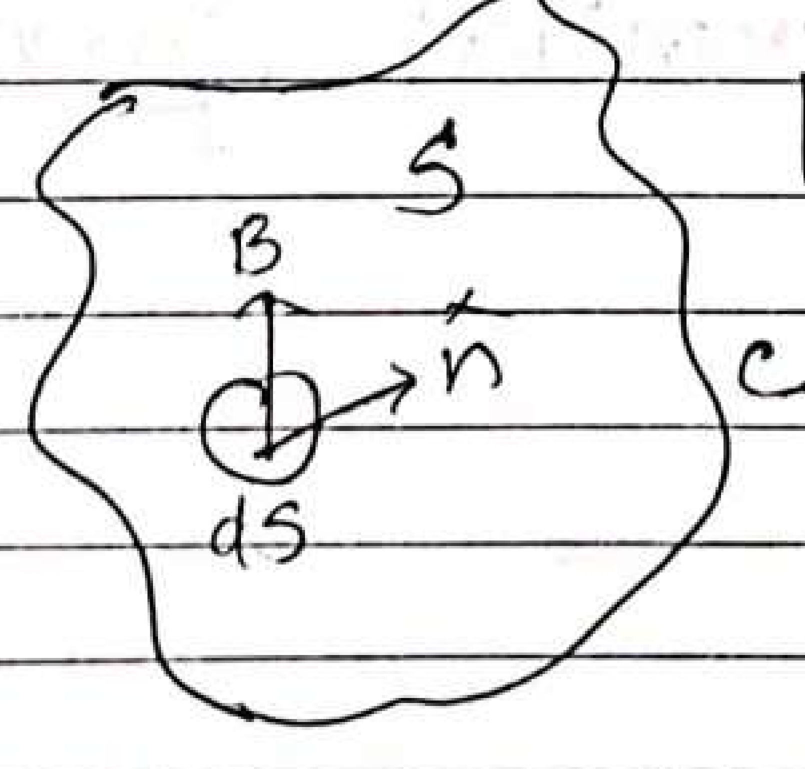
(1) In the fig, we have a closed surface S and a boundary C of it. The boundary forms a conducting path or circuit through which a uniform magnetic flux density B exists.

(2) According to Faraday's law, electromagnetic force induced in closed loop is negative rate of change of the magnetic flux.

$$e = -\frac{d\phi}{dt} \quad \text{——} \quad (1)$$

(3) Total magnetic flux on any arbitrary surface S is

$$\phi = \oint_S \vec{B} \cdot \vec{ds} \quad \text{——} \quad (2)$$



[Note :- Draw this fig after point no (1)]

$$\therefore \quad e = -\frac{d}{dt}\left[\oint_s B \cdot ds\right]$$

$$= -\oint_s \left[\frac{\partial B}{\partial t}\right] ds \quad \text{——} \quad (3)$$

(4) The electromotive force is the work done in carrying a unit charge around the closed loop

$$\therefore \quad e = \oint_l E \cdot dl \quad \text{—— (4)}$$

$$\therefore \quad \oint_l E \cdot dl = - \oint_s \left[\frac{\partial B}{\partial t}\right] ds$$

(5) By using Stoke's theorem line integral can be converted to surface integration as

$$\oint_l E \cdot dl = \oint_s (\nabla \times E) \cdot ds$$

$$\therefore \quad \oint_s (\nabla \times E) \cdot ds = - \oint_s \left[\frac{\partial B}{\partial t}\right] ds$$

$$\nabla \times E = - \frac{\partial B}{\partial t}$$

This is Maxwell's third equation in differential form.

Maxwell's Third Equation in Integral Form

(1) From differential form of Maxwell's Third equation, we have

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

(2) Integrating over entire surface on both sides

$$\oint_s (\nabla \times E)\, ds = \oint_s -\frac{\partial B}{\partial t}\, ds$$

using Stoke's theorem, convert surface integral to line integral

$$\oint_l E \cdot dl = -\frac{\partial}{\partial t} \oint_s \vec{B} \cdot \vec{ds}$$

This eqn is known as Maxwell's third equation in integral form.

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Maxwell's fourth Equation in Differential Form

(1) The line integral of the tangential component of the magnetic field over one complete path is equal to the current enclosed by the closed path

$$\oint_l B \cdot dl = \mu_0 I \quad \text{—— (1)}$$

we know $B = \mu_0 H$ and
$I = \oint_s \vec{J} \cdot d\vec{s}$ [where J is current density]

$$\therefore \oint_l \mu_0 \vec{H} dl = \mu_0 \oint_s J \cdot ds$$

$$\oint_l \vec{H} \cdot dl = \oint_s \vec{J} \cdot d\vec{s}$$

(2) using Stoke's theorem, to convert line integral to surface integral

$$\oint_s (\nabla \times H) ds = \oint_s \vec{J} \cdot ds$$

$$\therefore \nabla \times H = \vec{J} \quad \text{—— (2)}$$

(3) This eqn needs to be verified for time varying field as its validity holds for steady state only.
Because if it is correct then conservation of charge will be violated.
The reason is very simple; divergence of a curl is zero.

$$\vec{\nabla}.\vec{J} = \vec{\nabla}(\vec{\nabla}\times\vec{H}) = 0$$

This is in contrast with the continuity equation

$$\vec{\nabla}\vec{J} = -\frac{\partial \rho}{\partial t}$$

(4) To correct this, Maxwell suggested the total current density needs an additional component i.e. $\vec{J}'$.

(5) Now, if divergence is taken then using eqn (2)

$$\vec{\nabla}(\vec{\nabla}\times\vec{H}) = \vec{\nabla}(\vec{J} + \vec{J}')$$

$$0 = \vec{\nabla}(\vec{J} + \vec{J}')$$

$$\therefore \vec{\nabla}\vec{J} = -\vec{\nabla}\vec{J}'$$

$$\therefore -\vec{\nabla}\vec{J}' = -\frac{\partial \rho}{\partial t}$$

$$\therefore \vec{\nabla}\vec{J}' = \frac{\partial \rho}{\partial t}$$

(6) But from Maxwell's 1st equation

$$\vec{\nabla}.\vec{D} = \rho$$

$$\therefore \vec{\nabla}\vec{J}' = \frac{\partial (\vec{\nabla}.\vec{D})}{\partial t} = \nabla\frac{\partial \vec{D}}{\partial t}$$

Hence $\vec{J}' = \frac{\partial \vec{D}}{\partial t}$

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(7) Therefore Maxwell's Fourth Equation is

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

The additional term $\frac{\partial \vec{D}}{\partial t}$ is called Maxwell's correction and it is known as displacement current.

Maxwell's Fourth Equation in Integral Form:

(1) using differential Form of Maxwell's Fourth eqn.

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Taking Surface integration on both the sides.

$$\oint_S (\vec{\nabla} \times \vec{H}) . \vec{ds} = \oint_S \left(J + \frac{\partial D}{\partial t} \right) ds$$

Now using Stoke's theorem to convert surface integral into line integral

$$\therefore \oint_l \vec{H} . \vec{dl} = \oint_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \vec{ds}$$

This is Maxwell's Fourth eq^n in integral form.

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Question's to be Studied.

1. Explain physical significance of Divergence of a vector.
2. Explain physical significance of curl of a vector.
3. Derive Maxwell's 1st, 2nd, 3rd, 4th equation in Differential form and integral Form
4. Show that divergence of curl of a vector is zero.
5. Sums on Gradient of a scalar, Divergence of a vector and curl of a vector